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AFC@MESA LAB

FRACTIONAL ORDER MECHANICS: WHY, WHAT AND WHEN

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Oct. 30, 2012. Tuesday 11:00-12:00
Castle Research Facility Room #22

MESA LAB

http://mechatronics.ucmerced.edu

- **Mechatronics, Embedded Systems and Automation**
  - **Backup name: *Mechatronics, Energy Systems and Autonomy***
  - **ASME DED, MESA TC. http://iel.ucdavis.edu/mesa/**
    - **2013 MESA conference: Portland, OR http://www.asmeconferences.org/IDETC2013/**

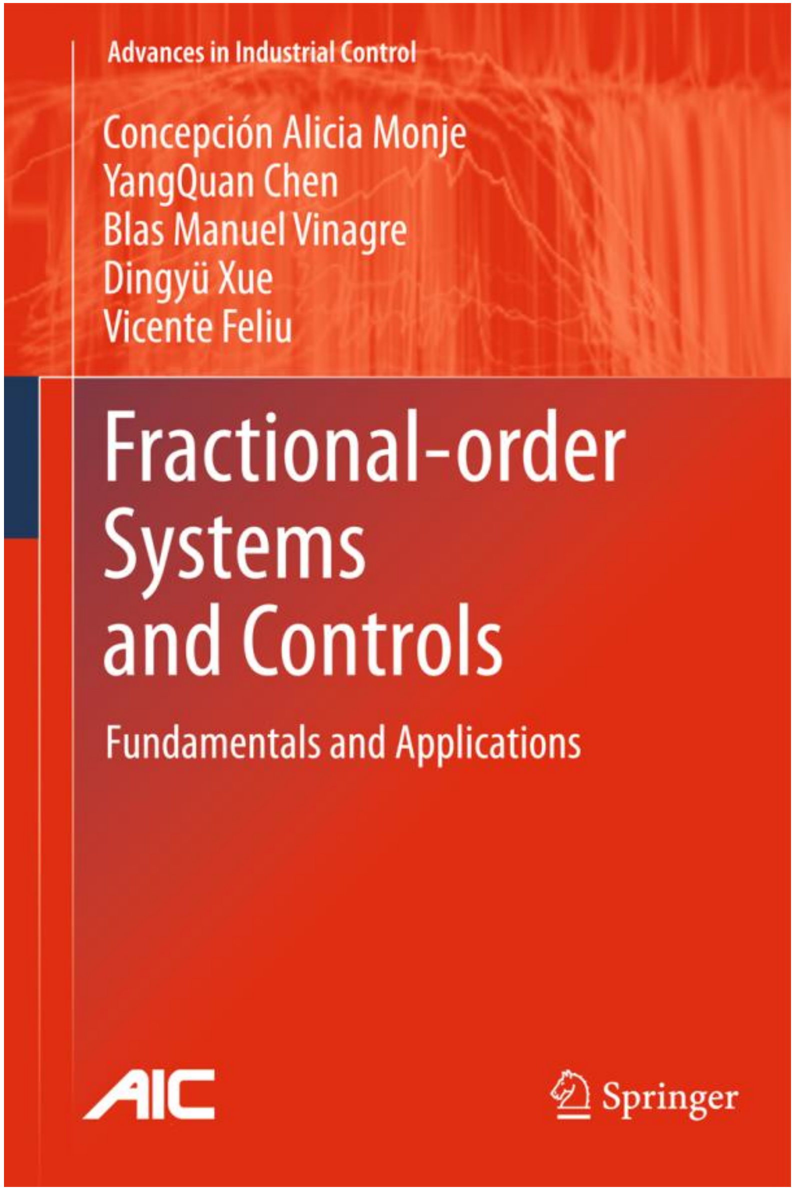
MESA Lab Philosophy and Ambition

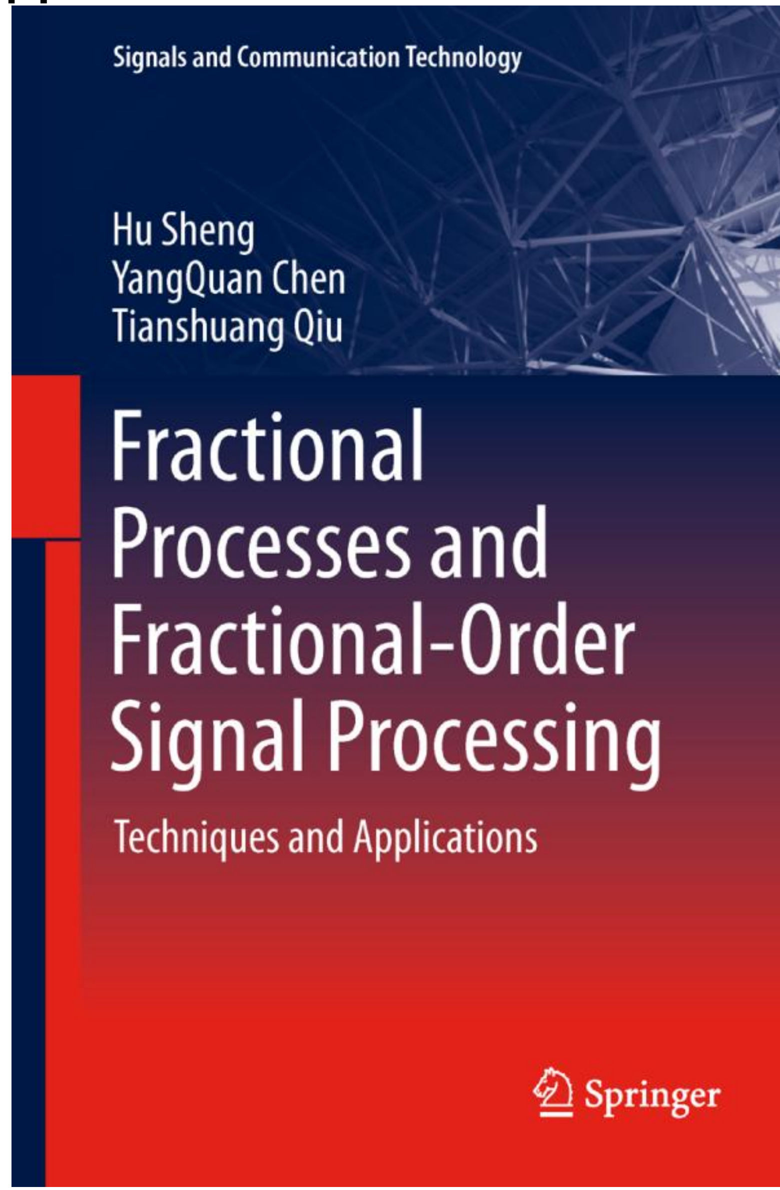
- ***"We make real systems that work and others want them."***
- **MESA Lab: Staying on top and for sustainability.**
- Nationally and internationally visible and prominent!

MESA Research Areas/Strengths

- Unmanned Aerial Systems and UAV-based Personal Remote Sensing (PRS)
- Cyber-Physical Systems (CPS)
- Modeling and Control of Renewable Energy Systems
- Mechatronics
- **Applied Fractional Calculus (AFC)**

FOMech: WHY?

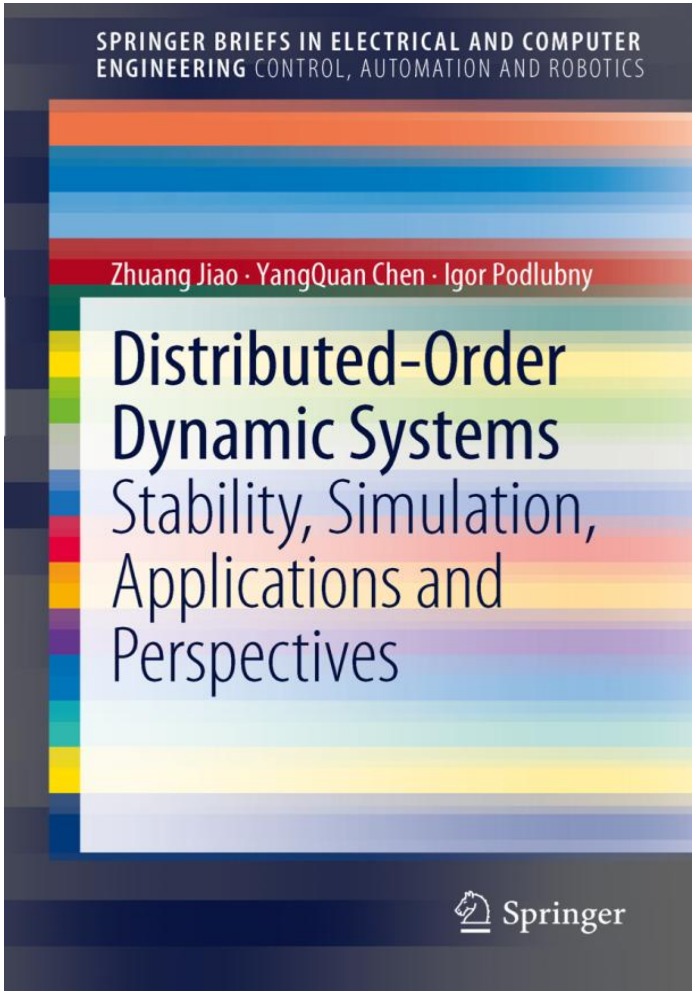


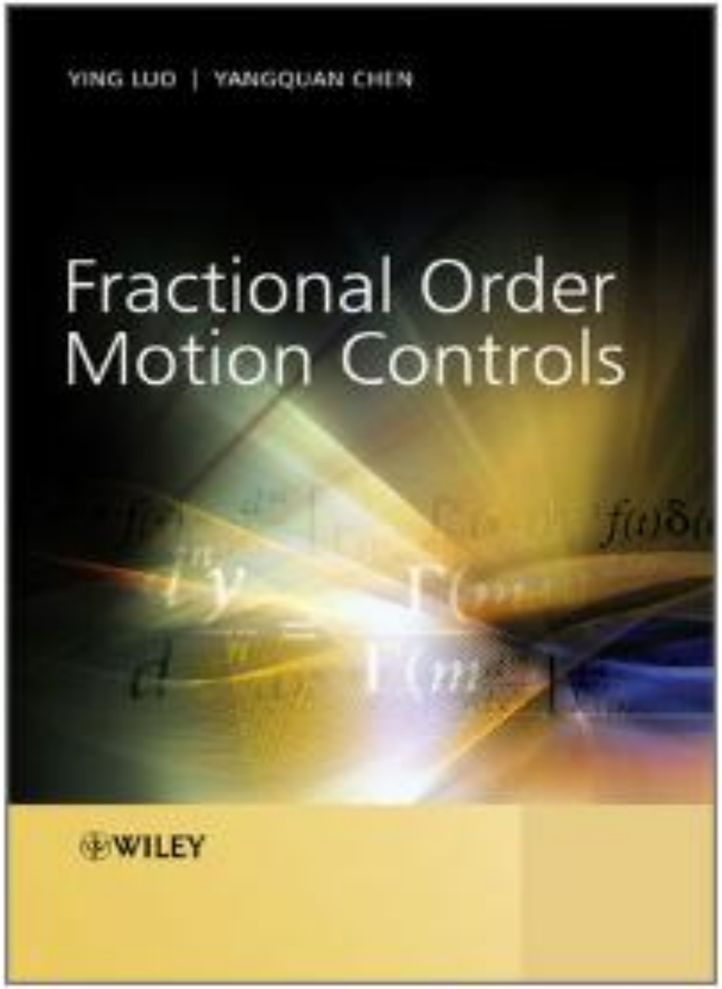


2001-2010

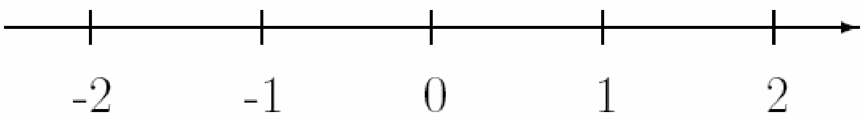
2005-2011

FOMech: WHY?


SPRINGER BRIEFS IN ELECTRICAL AND COMPUTER ENGINEERING CONTROL, AUTOMATION AND ROBOTICS
Zhuang Jiao · YangQuan Chen · Igor Podlubny
Distributed-Order Dynamic Systems
Stability, Simulation, Applications and Perspectives
Springer


YING LUO | YANGQUAN CHEN
Fractional Order Motion Controls
WILEY

… from integer to non-integer …



$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n}$$

$$x^n = e^{n \ln x}$$

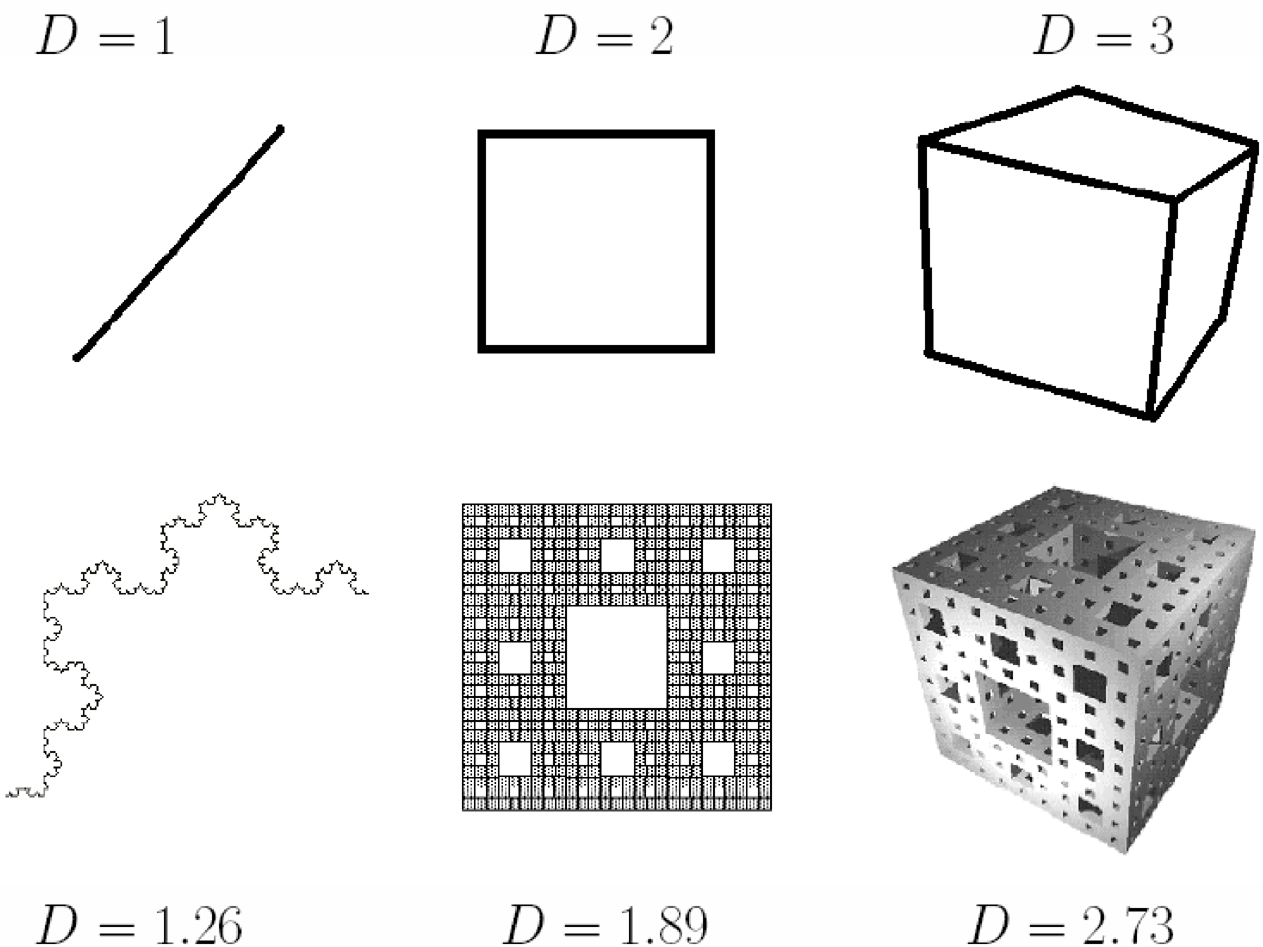
$$n! = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot (n-1) \cdot n,$$

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \qquad x > 0,$$

$$\Gamma(n+1) = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot n = n!$$

Slide credit: Igor Podlubny

... from integer to non-integer ...



Slide credit: Igor Podlubny

Interpolation of operations

$$f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \frac{d^3 f}{dt^3}, \quad \ldots$$

$$f, \quad \int f(t)dt, \quad \int dt \int f(t)dt, \quad \int dt \int dt \int f(t)dt, \quad \ldots$$

$$\ldots, \quad \frac{d^{-2} f}{dt^{-2}}, \quad \frac{d^{-1} f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \ldots$$

Slide credit: Igor Podlubny

"Fractional Order Thinking" or, "In Between Thinking"

- For example
  - Between integers there are non-integers;
  - Between logic 0 and logic 1, there is the "**fuzzy logic**";
  - Between integer order splines, there are "**fractional order splines**"
  - Between integer high order moments, there are **noninteger order moments (e.g. FLOS)**
  - Between "integer dimensions", there are **fractal dimensions**
  - Fractional Fourier transform (FrFT) – in-between time-n-freq.
  - Non-Integer order calculus (fractional order calculus – abuse of terminology.) (FOC)

Conclusion of Talk

Integer-Order Calculus

Fractional-Order Calculus

Slide credit: Richard L. Magin, ICCC12

Fractional Calculus was born in 1695

$$\frac{d^n f}{dt^n}$$

What if the order will be $n = ½$?

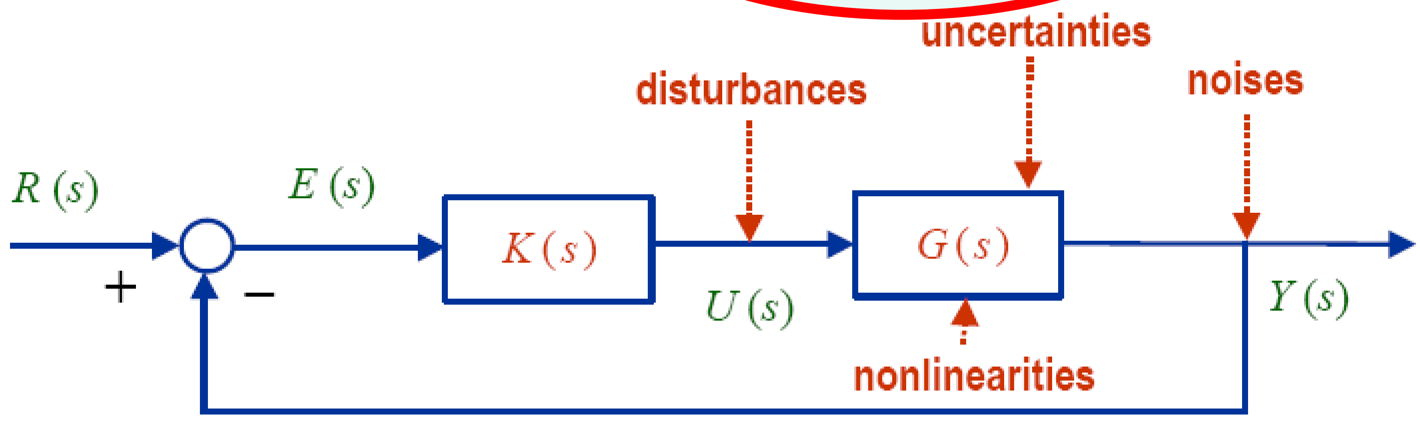
It will lead to a paradox, from which one day useful consequences will be drawn.

G.F.A. de L'Hôpital
(1661–1704)

G.W. Leibniz
(1646–1716)

FOMs and Fractional Order Controls

- IO Controller + IO Plant
- FO Controller + IO Plant
- FO Controller + FO Plant
- IO Controller + FO Plant



Concepcin A. Monje, YangQuan Chen, Blas Vinagre, Dingyu Xue and Vicente Feliu (2010). "Fractional Order Systems and Controls - Fundamentals and Applications." Advanced Industrial Control Series, Springer-Verlag, www.springer.com/engineering/book/978-1-84996-334-3 (2010), 415 p. 223 ill.19 in color.

Rule of thumb for Fractional Order Thinking

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- *1/f* ^{a} noise

- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

Fractional Order Mechanics: WHY?

- Softmatter / hardmatter
- Softbody / Rigidbody
- Lumped / distributed
- Granular, particulate, porous, disordered … materials
- …

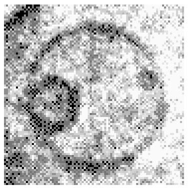
Soft matter？

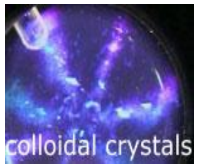
- Soft matters, also known as ***complex fluids***, behave unlike ideal solids and fluids.
- ***Mesoscopic*** macromolecule rather than microscopic elementary particles play a more important role.

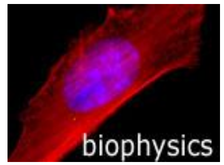
Typical soft matters

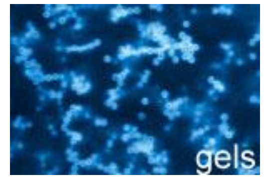
- Granular materials
- Colloids, liquid crystals, emulsions, foams,
- Polymers, textiles, rubber, glass
- Rock layers, sediments, oil, soil, DNA
- Multiphase fluids
- Biopolymers and biological materials

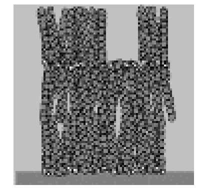
highly deformable, porous, thermal fluctuations play major role, highly unstable

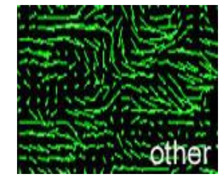


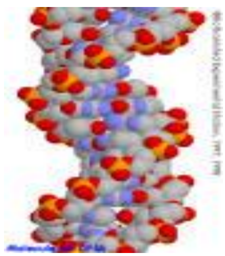


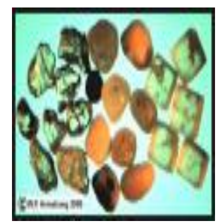


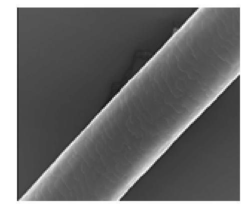


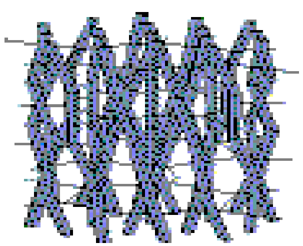


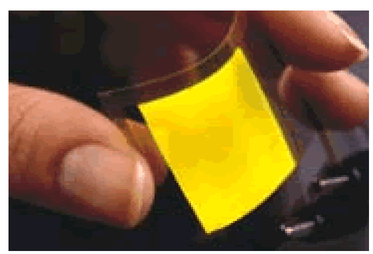


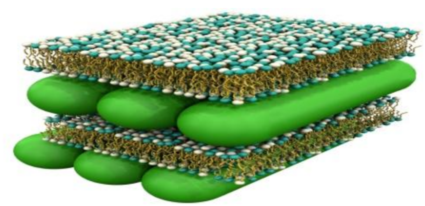












Slide credit: Wen Chen of HHU

Constitutive relationships

- **Hookian law in ideal solids**: $F = kx$
- **Ideal Newtonian fluids**: $F = \upsilon \frac{\partial u}{\partial y}$
- **Newtonian 2nd law for rigid solids**: $F = m \frac{d^2 x}{dt^2}$
- **One model of soft matter:** $F = \rho \frac{\partial^\alpha x}{\partial t^\alpha} \quad 0 \le \alpha \le 2$

Slide credit: Wen Chen of HHU

Fractional Order Mechanics!

Hooke's law: $F = kx$

Newton's fluid: $F = kx'$

Newton's 2nd law: $F = kx''$

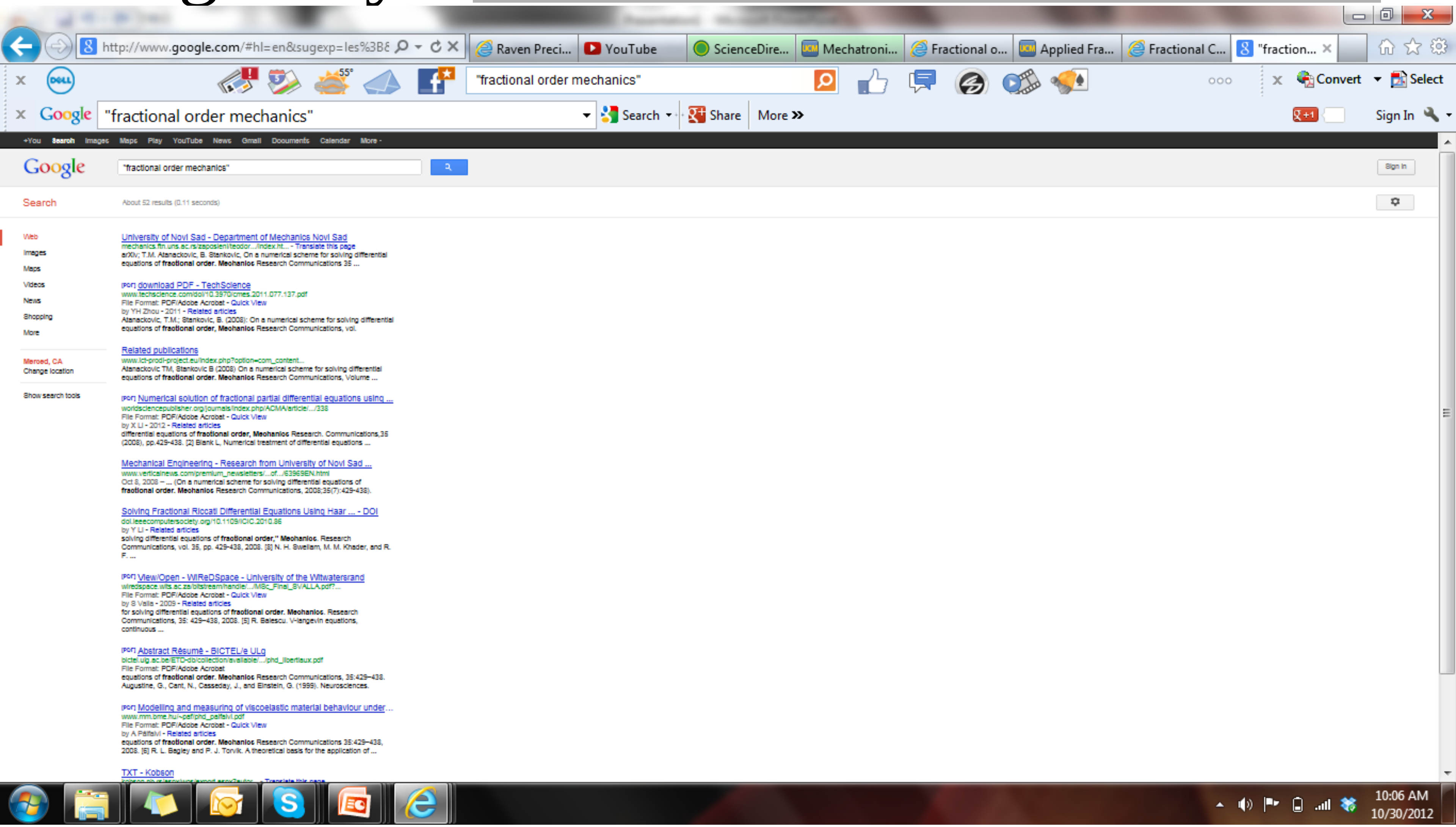
$$F(t) = k\,x^{(\alpha)}(t)$$

Going in-between: interpolation of operators:

$$\cdots,\quad \frac{d^{-2}f}{dt^{-2}},\quad \frac{d^{-1}f}{dt^{-1}},\quad f,\quad \frac{df}{dt},\quad \frac{d^2f}{dt^2},\quad \cdots$$

- Kurt Lewin: “There is nothing so practical as good theory" (p. 169).
  - Lewin, K. (1951). Field theory in social science. New York: Harper & Row.

Google says "Fractional Order Mechanics"



G.W. Scott Blair (1950)

- “We may express our concepts in Newtonian terms if we find this convenient but, if we do so, we must realize that we have made a translation into a language which is foreign to the system which we are studying.”

Key reference links

- CDC10 tutorial: http://mechatronics.ece.usu.edu/foc/cdc10tw/
- http://people.tuke.sk/igor.podlubny/USU/
- http://mechatronics.ece.usu.edu/foc/afc/
- http://www.wydawnictwa.pcz.pl/book/102/introduction-fractional-mechanics
- FDA Express: http://em.hhu.edu.cn/fda/
- ICFDA2014 http://www.icfda14.dieei.unict.it/committee.html

FOMech:
What should be included in the course?

- At UC Merced, ME280 “Fractional Order Mechanics” will be officially offered from Fall 2013 by Prof. YangQuan Chen. (was VOM)
- We are defining this course right now with some initial thinking

TOC of ME280 FOMech

- FC basics; FC signals and systems
- FC modeling (ML fitting) of complex relaxation processes
- *Bagley-Torvik* mechanics
- Fractional Euler-Lagrange Equation
- Advanced topics (application oriented) (FISP – focused independent study and presentation)
  - Battery system models, biological signal processing
  - Fractional Order ESC, nanomaterial modeling
  - Salinity dynamics, complexity quantification
  - Hysteresis modeling and compensation
  - FO stochastic mechanics for evolving complex networks etc.

FOMech: WHEN?

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- $1/f^{a}$ noise
- Porous media
- Particulate
- Granular
- Lossy
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- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …