Simulating shear thickening fluids with fractional derivatives

Eric Brown, Oktar Ozgen, Marcelo Kallman
University of California, Merced

Fractional Calculus Day
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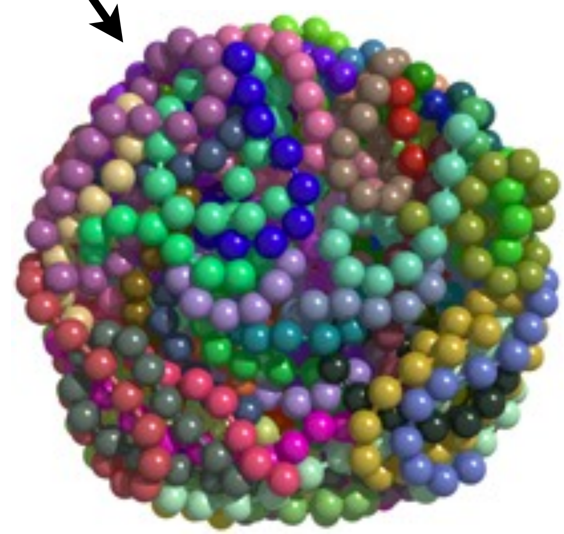
Using novel properties of soft materials for mechanical control

- jamming of granular materials for simplified control of robotic gripping
- entangled granular materials for highly deformable, high strength structures
- shear thickening fluids for impact resistance
  - modeled with fractional calculus

BAE Systems

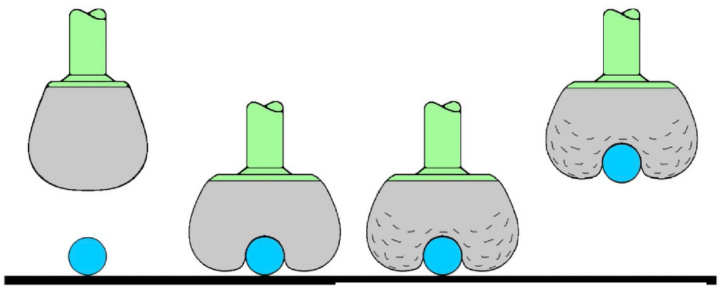


Applying granular materials to robotic gripping: picking up arbitrary objects

- control problem: many joints, sensing, feedback required

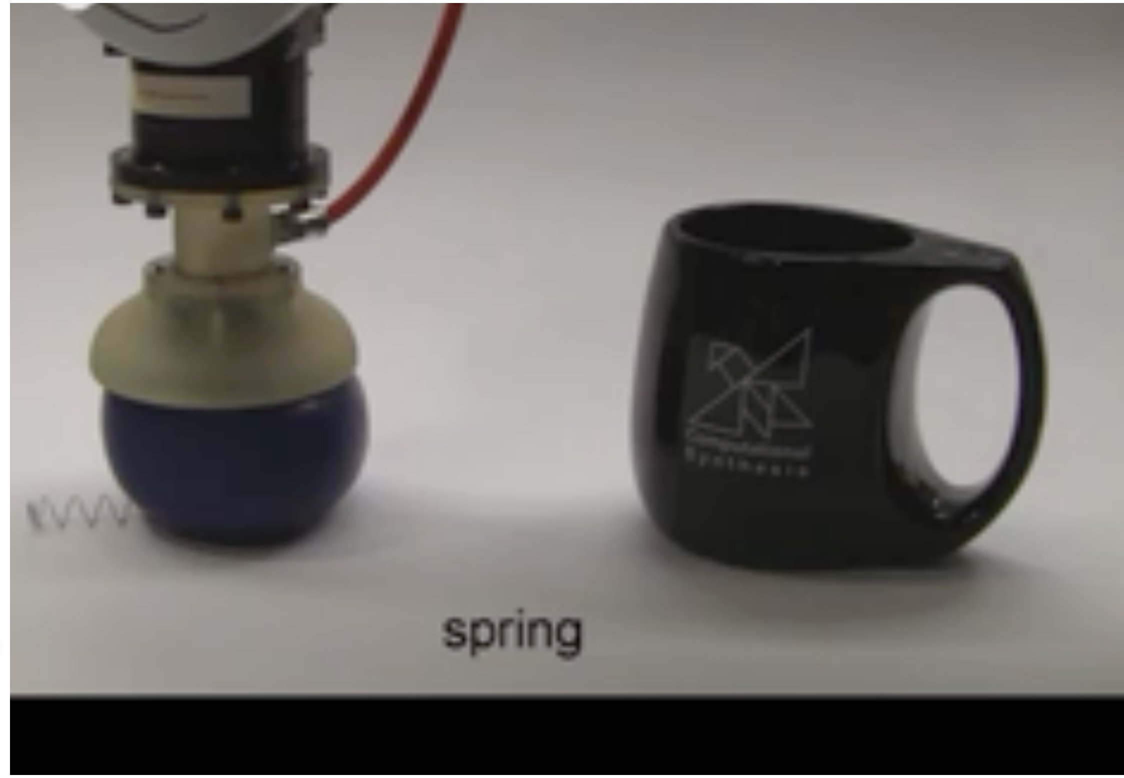
- can a high level of control be achieved with fewer actuators?

Solution: Controllable jamming transition of granular materials



fluid → jammed

air evacuated



video by John Amend

advantages of controllable jamming transition:

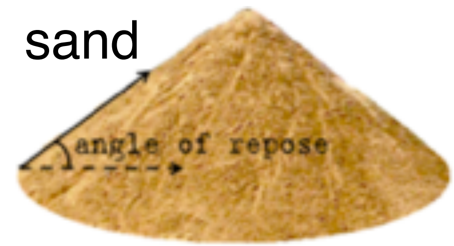
- forms to shape of object
- distributes stress
- no external computation/feedback required
- transition and gripping strength controllable by vacuum pump

Brown et al. PNAS 107(43), (2010)

Using entanglement of granular materials for highly deformable, high strength structures

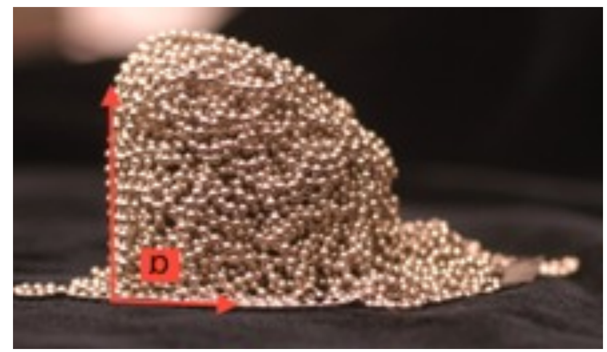
Granular material strength usually limited by confining stress
=> unconfined piles are weak

sand



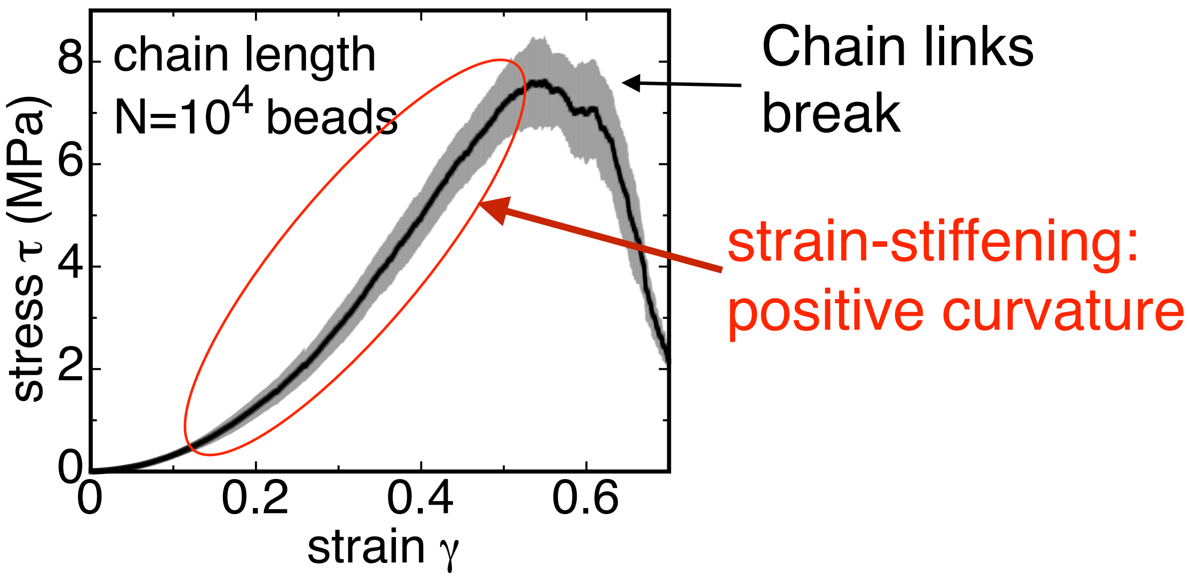
chains

Packings of entangled particles can support a load without confinement:

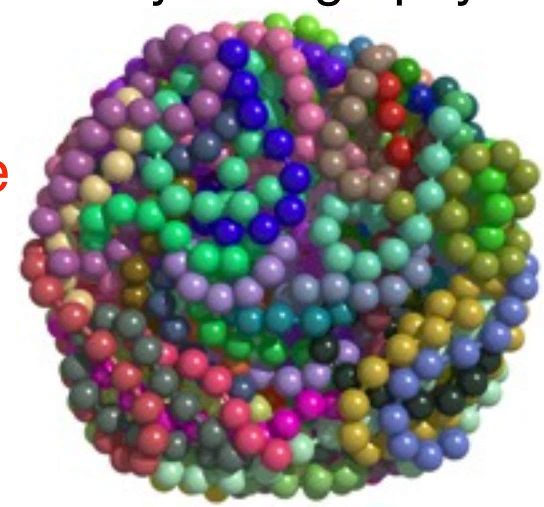


• material properties can be tuned by modifying connectivity (similar to polymers)
• structure and strength can be controlled by packing and loading history
• applications: pourable foundations, shapeable ‘granular rubbers’

Strong strain-stiffening occurs when chains form system-spanning clusters of entanglements



reconstruction of entanglements from x-ray tomography



- entanglement of long chains produces ultimate strengths ~10^3 times stronger than unconfined granular packings
  - -entanglements prevent failure from shear banding

Brown, Nasto, Athanassiadis, Jaeger, Phys. Rev. Lett. 108, 108302

Shear thickening Fluid phenomena

suspension of cornstarch in water

- fluid-like at rest or under low stress
- supports weight temporarily like a solid in response to high stress

Shear thickening Fluid phenomena

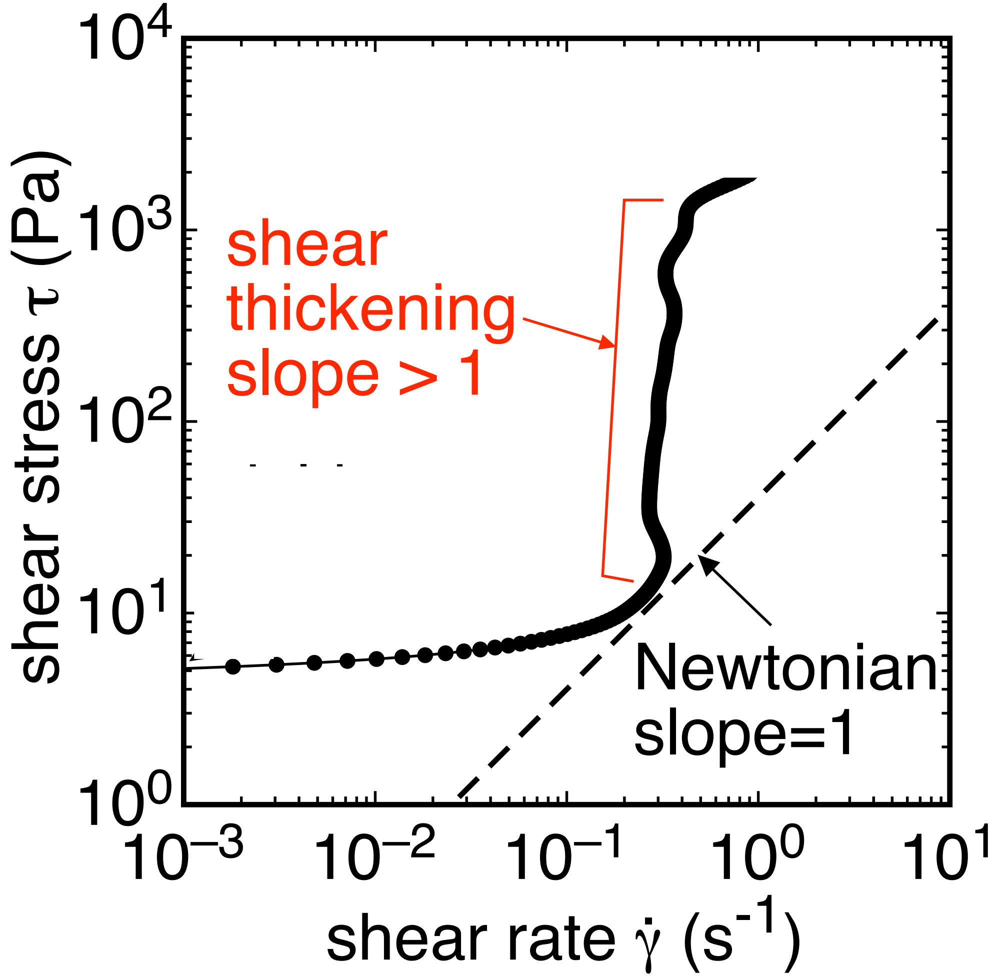
- object can bounce off of solid surface
- material “melts” into fluid state after stress reduced

Question:
What causes these transient solid-like properties of shear thickening fluids?

- ability to run on surface
- impact resistance
- bouncing

Rheology characterized by steady state stress vs. shear rate $\tau(\dot{\gamma})$

cornstarch in water (~50/50)



Problem:

Models of shear thickening based on steady state stress vs. shear rate can't explain:

- high stresses (i.e. enough to support a person ~10^5 Pa)
- bouncing

- Hydroclusters (Brady and Bossis, 1985)
  - lubrication: breaks down before such large effective viscosities could be reached (~10^2 Pa)
- Dilatancy (Brown and Jaeger, 2012)
  - surface tension: explains steady state $\tau(\dot{\gamma})$
  - not strong enough to support such large stresses (~10^3 Pa)
- Added mass (Waitukaitis and Jaeger, 2012)
  - added mass: requires very higher speeds and more available fluid mass than are found in most observations (~10^4 Pa)

What is missing from these models?

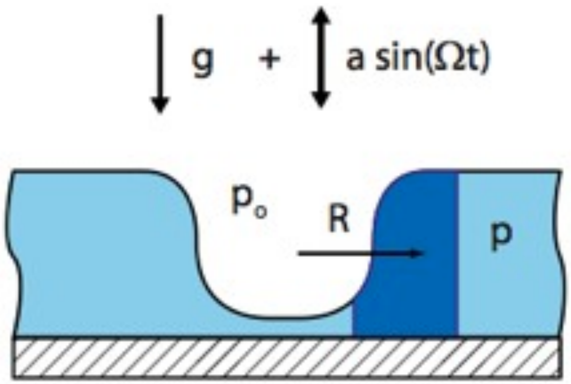
- hysteresis
- solid stiffness

=>fractional calculus

A phenomena we understand:
Stable fingers under vibration of a shear thickening fluid layer

- localized structures stable and evolve over many cycles of vibration

Stability of structures due to Hysteresis

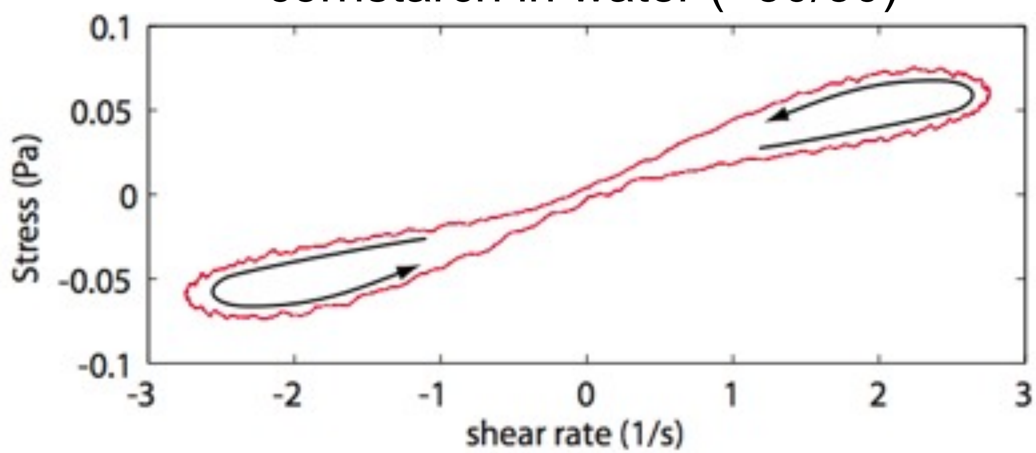


For any function $\tau(\dot{\gamma})$ without hysteresis

=>local surface deformations unstable due to gravity

=> melt into a flat fluid layer

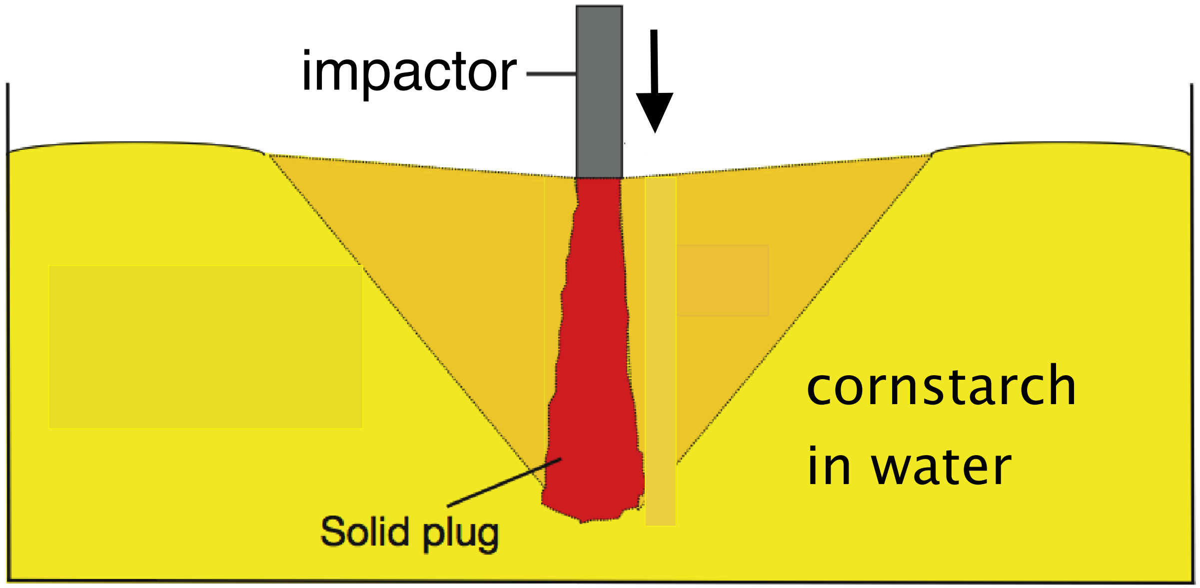
cornstarch in water (~50/50)



Hysteresis in $\tau(\dot{\gamma})$ allows net force over cycle to balance gravity
=> structures survive over many cycles
=> hysteresis can be modeled with fractional derivatives

Deegan, 2010

Something we suspect:
Transient solid-like plug forms in front of impact



A temporary solid-like structure could hold weight

=> suggests modeling with an effective material stiffness which is higher after a high velocity impact (=>hysteresis)

Waitukaitus and Jaeger, 2012

Our Model for shear thickening fluids

- Viscosity → Smoothed Particle Hydrodynamics
  - computationally efficient hydrodynamic calculations
- Elasticity → springs between nearby particles
- Plasticity → Adaptive spring relaxation length
- Hysteresis → Fractional derivatives
  - Fractional derivatives used to model combination of elastic (1st order) and viscous (2nd order) effects: [Germant 1938, Bagley 1983, Makris 1993, Mainardi 2010, and many others]

O. Ozgen, E. Brown, M. Kallmann. **Simulating Shear Thickening Fluids.** submitted to SIGGRAPH (2013)

Elasticity & Plasticity

- Nearby particles are connected by springs with stiffness k_{min}

$$F = -k_{min}\, \boldsymbol{x},$$

- The spring rest length is updated if the deformation **x** exceeds a threshold (models plasticity).

History Effects from fractional derivatives

$$F = -k_{min}\,\boldsymbol{x} - k_{hist}\,\|D^q x\|\,\boldsymbol{x}.$$

elastic term ↗

↑ fractional derivative: "stiffness" dependent on velocity history (larger after high speed impact)

↓

calculated as sum over velocities of n previous timesteps Δt

↓

$$F = -1\left(k_{hist}\,\frac{\Delta t^{1-q}}{\Gamma(3-q)}\sum_{p=0}^{n} a_p \boldsymbol{v}_p + k_{min}\right)\boldsymbol{x},$$

- fractional derivative order q=1/2
- weight a_p decreases with increasing p
- Γ = Gamma function

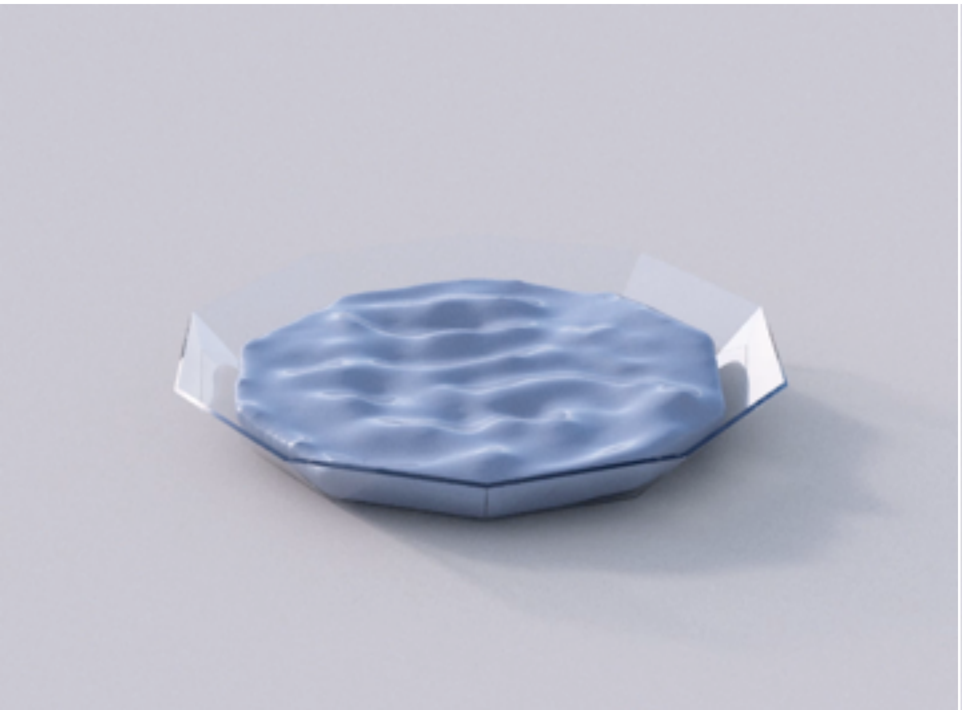
Simulation of vibrated layer

real world

simulation

Stable fingers under vibration with fractional derivatives

without fractional derivatives ($k_{hist}=0$) with fractional derivatives



- hysteresis allows localized structures to be stable
- For $\tau(\dot{\gamma})$ without hysteresis, such structures would be unstable due to gravity and melt into a flat fluid layer (Deegan 2010)

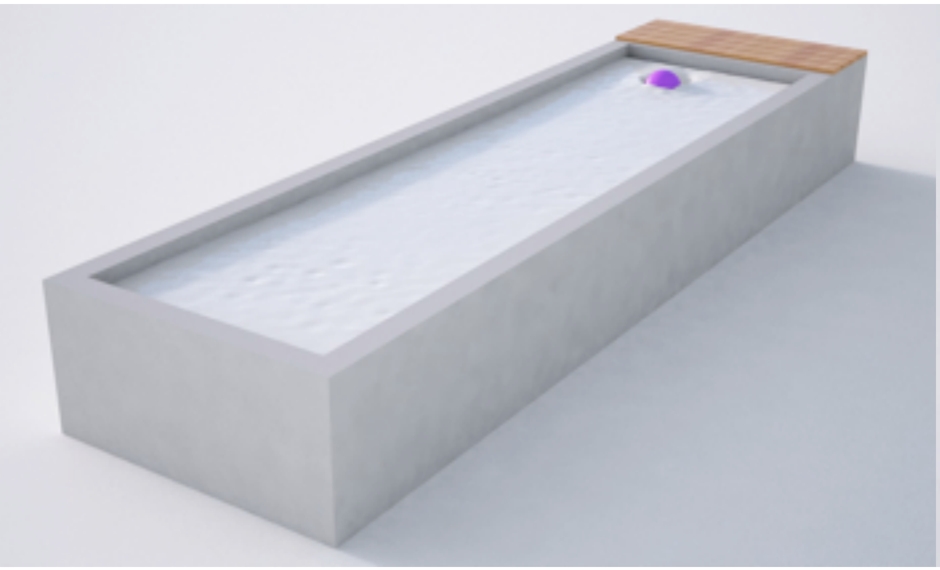
Simulation of ball rolling on surface

real world

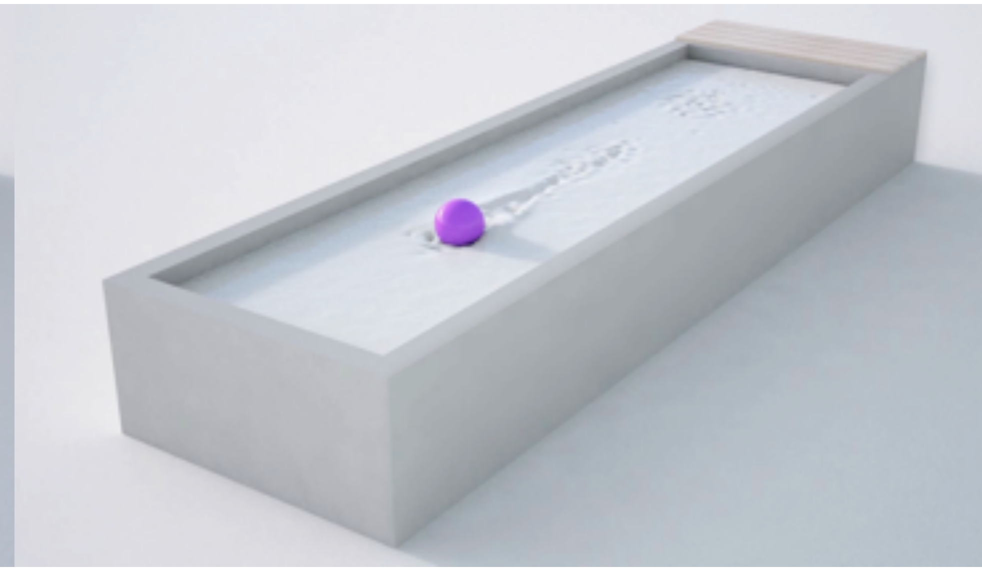
simulation

Supporting weight with fractional derivatives

without fractional derivatives



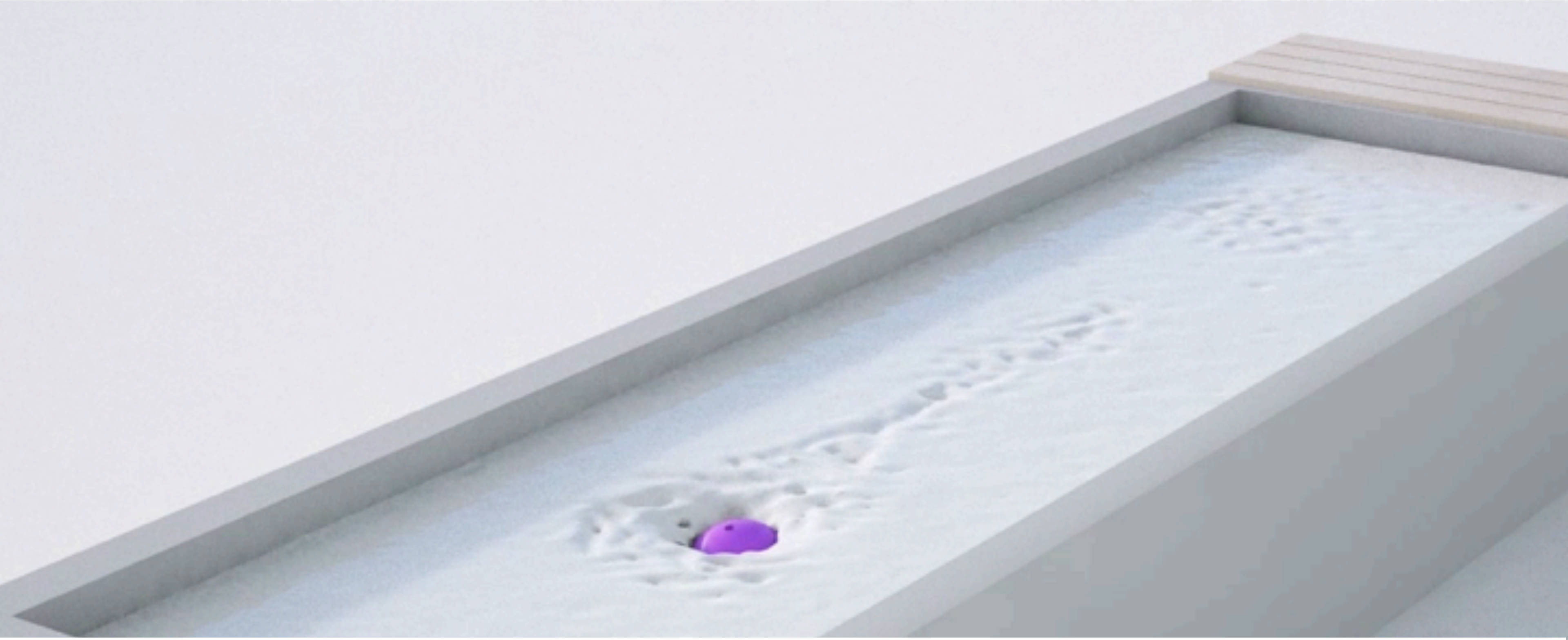
with fractional derivatives



• fractional derivative term supports weight if there is a recent history of high speed

- models temporary solid structure (Waitukaitus & Jaeger,2012)

Oscillations in sinking speed due to hysteresis



- For $\tau(\dot{\gamma})$ without hysteresis, ball would monotonically approach terminal velocity [von Kann et al. (2011)]

Conclusions

- First Fractional Calculus applications in Computer Graphics for shear thickening fluid simulation
- First simulation of any kind of phenomena to produce phenomena associated with shear thickening:
  - stable fingers under vibration
  - supporting weight and bouncing (like a solid)
  - oscillations in velocity of sinking object
- Simulations confirm hysteresis can explain these phenomena, rather than shear thickening stress vs. shear rate function

O. Ozgen, E. Brown, M. Kallmann. **Simulating Shear Thickening Fluids.** submitted to SIGGRAPH (2013)

Applications: tunable impact resisting materials

- Next steps: determine what controls the effective stiffness k_{hist}
  - intrinsic material properties (i.e. particle stiffness)
  - boundary conditions
- dynamic material response can be tuned by changing particles, liquids, packing density, or making composites

BAE Systems

D30

Students and collaborators:

granular gripper:
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John Amend (Cornell)
Prof. Hod Lipson (Cornell)
Erik Steltz (iRobot)
Annan Mozeika (iRobot)
Heinrich Jaeger (University of Chicago)

entangled granular materials:
Alice Nasto (MIT)
Thanasi Athanassiadis (University of Chicago)

shear thickening:
Oktar Ozgen (UC Merced)
Prof. Marcelo Kallman (UC Merced)
Ben Allen (UC Merced)

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