ME280 "Fractional Order Mechanics"

Fall 2013, Instructor: YangQuan Chen (ychen53@ucmerced.edu)

Catalog Description: This course prepares students with fractional calculus (differentiation or integration of non-integer order) and fractional dynamic modeling of complex mechanical systems such as porous medias, particulate systems, soft matters etc. that have inherent nature of memory, heredity, or long-range dependence (LRD), or long range interactions at or across various scales.

Textbooks:

[1] Igor Podlubny (1999). "*Fractional Differential Equations. An Introduction to Fractional Derivatives, Fractional Differential Equations, Some Methods of Their Solution and Some of Their Applications*", Academic Press, San Diego - New York – London. ISBN-13: 978-0125588409

[2] Richard Magin (2006). "*Fractional Calculus in Bioengineering*" Begell House Publishers. ISBN-13: 978-1567002157

**Course Learning Outcomes:** Upon completion of ME280, students should be able to

1. Perform basic fractional calculus (FC) math derivations;
2. Do fractional order modeling of relaxation processes from complex systems using both numerical simulation as well as **lab experiments** on complex system components such as *super-capacitor charging process* and *artificial muscles*;
3. Do numerical solution of fractional order differential equations;
4. Perform simple fractional order damping control analysis;
5. Understand the fractional mechanics in classical sense (Bagley-Torvik's stress-strain relationships etc.)
6. Appreciate the new tool of fractional order calculus of variation and its role in fractional order Euler-Lagrange mechanics
7. Acquire distributed-order thinking in fractional order mechanics.
8. Use the theory and techniques in "fractional order mechanics" to address the modeling of non-mechanical systems in their respective research fields such as thermal/fluid systems, bioengineering system, cognitive science, material science, biological systems, physiological systems, networked systems, human-centric man-made systems etc.
9. Agree that "*being anomalous is normal*" when fractional calculus (FC) is used as the modeling tool.

Prerequisites: Instructor approval (via email). ODE, Laplace transform, basic modeling techniques etc.

Class size: 10. Send email request for enrollment first. Graduate students only.

Course Outline:

1) Motivations and real world needs (soft-matter, bioengineering, single molecule dynamics, etc); (2 weeks)
2) Mathematical foundations of fractional calculus; (2 weeks)
3) Fractional order system modeling; (2 weeks)
4) Fractional mechanics in classical sense (Bagley-Torvik) (4)
5) Fractional order damping control (1 week)
6) Fractional Euler-Lagrange mechanics and fractional variational principle. (2 weeks)
7) Distributed-order mechanics (1 week)
8) Focused Independent Study and Presentation (**FISP**) (2 weeks, max. 12 talks with 25 min. each) **Sample Topics:** Battery system models, biological and physiological signal processing, fractional order extremum seeking control, nanomaterial modeling of particles with long range interactions, extreme event dynamics, complexity quantification, hysteresis modeling and compensation, fractional order stochastic mechanics for evolving complex networks, controlled cognition etc.

Integer-Order Calculus Fractional-Order Calculus
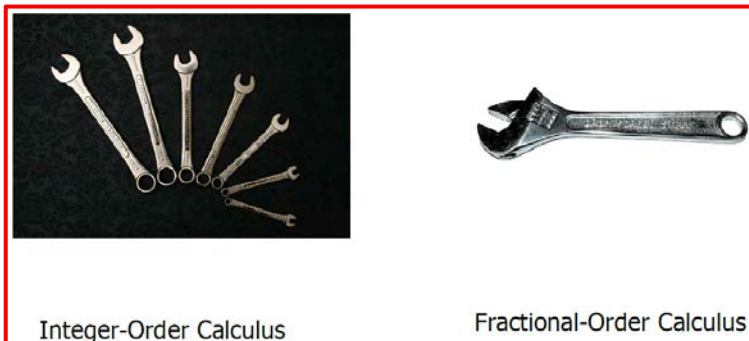
Fractional Order Mechanics!

Hooke's law: $F = kx$

Newton's fluid: $F = kx'$ $\rightarrow F(t) = k\, x^{(\alpha)}(t)$

Newton's 2nd law: $F = kx''$

Going in-between: interpolation of operators:

$$\ldots, \quad \frac{d^{-2}f}{dt^{-2}}, \quad \frac{d^{-1}f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2f}{dt^2}, \quad \ldots$$

For more information, please visit http://www.asmeconferences.org/IDETC2013/Workshops.cfm (click **W6**)

MESA LAB @ UCMERCED

MECHATRONICS, EMBEDDED SYSTEMS AND AUTOMATION LAB

http://mechatronics.ucmerced.edu

APPLIED FRACTIONAL CALCULUS (AFC)

... from integer to non-integer ...

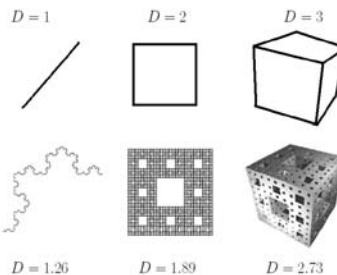$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n}$$

$$x^n = e^{n \ln x}$$

$$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n,$$

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \quad x > 0,$$

$$\Gamma(n+1) = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n = n!$$

... from integer to non-integer ...

$D = 1$ $D = 2$ $D = 3$

$D = 1.26$ $D = 1.89$ $D = 2.73$

Interpolation of operations

$$f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \frac{d^3 f}{dt^3}, \quad \ldots$$

$$f, \quad \int f(t)dt, \quad \int dt \int f(t)dt, \quad \int dt \int dt \int f(t)dt, \quad \ldots$$

$$\ldots, \quad \frac{d^{-2} f}{dt^{-2}}, \quad \frac{d^{-1} f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \ldots$$

Fractional Calculus was born in 1695

$\frac{d^n f}{dt^n}$

What if the order will be $n = \frac{1}{2}$?

It will lead to a paradox, from which one day useful consequences will be drawn.

G.F.A. de L'Hôpital (1661–1704)

G.W. Leibniz (1646–1716)

The beginning of a new stage

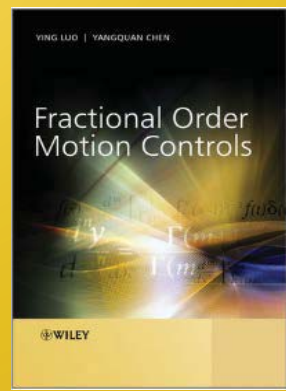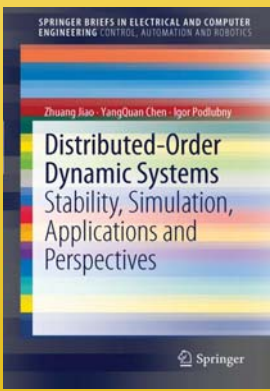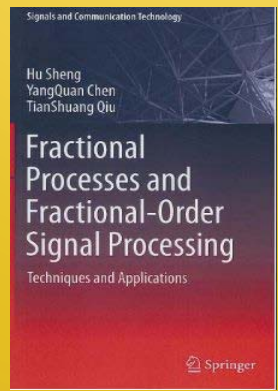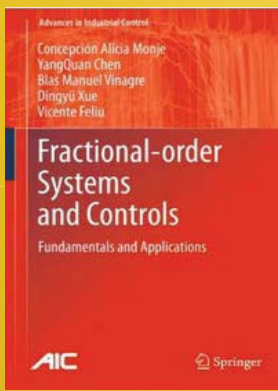| 1695	1960s	**You are here**
static models	dynamical models	fractional order modeling
geometry, algebra	differential and integral calculus	*Do better than* fractional calculus *the best doable before!*

FO derivative of sine wave
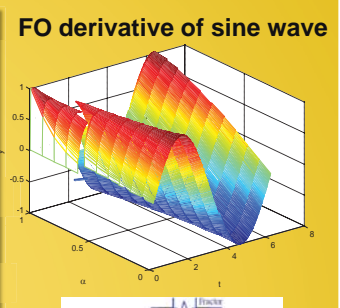
Rule of thumb

- Power law, Scale-free, scale invariant
- Heavy tailedness, fat tail
- Long range dependence (LRD)
- Porous media, Anomaly
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

"Go fractional!" – YangQuan Chen

7/13/1865. "Go west, young man." – Horace Greeley (1811-1872)

$$G(s) = -\frac{K}{R(s\tau)^\lambda}$$

Fractor

Fractional-order Systems and Controls
Fundamentals and Applications
Concepción Alicia Monje, YangQuan Chen, Blas Manuel Vinagre, Dingyü Xue, Vicente Feliu
Advances in Industrial Control
AIC   Springer

Fractional Processes and Fractional-Order Signal Processing
Techniques and Applications
Hu Sheng, YangQuan Chen, TianShuang Qiu
Signals and Communication Technology
Springer

Distributed-Order Dynamic Systems
Stability, Simulation, Applications and Perspectives
Zhuang Jiao · YangQuan Chen · Igor Podlubny
SPRINGER BRIEFS IN ELECTRICAL AND COMPUTER ENGINEERING CONTROL, AUTOMATION AND ROBOTICS
Springer

Fractional Order Motion Controls
YING LUO | YANGQUAN CHEN
WILEY

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