Turkish National Meeting on Automatic Control
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A Tutorial on Fractional Order Motion Control

Part VI: Effects of FO Controls on Nonlinearities

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- Fractional Order Motion Controls

John Wiley & Sons, Inc.

Hardcover, 454 pages, December 2012

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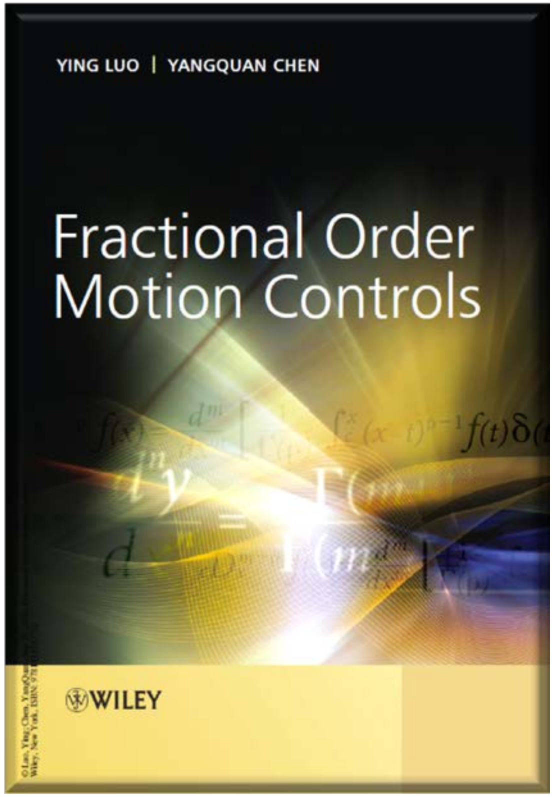

Scope of the book

Scope of the book

FO PID on DC Motor with Elastic Shaft

Proceedings of the 2006 American Control Conference
Minneapolis, Minnesota, USA, June 14-16, 2006
ThB11.5

Fractional Order PID Control of A DC-Motor with Elastic Shaft: A Case Study

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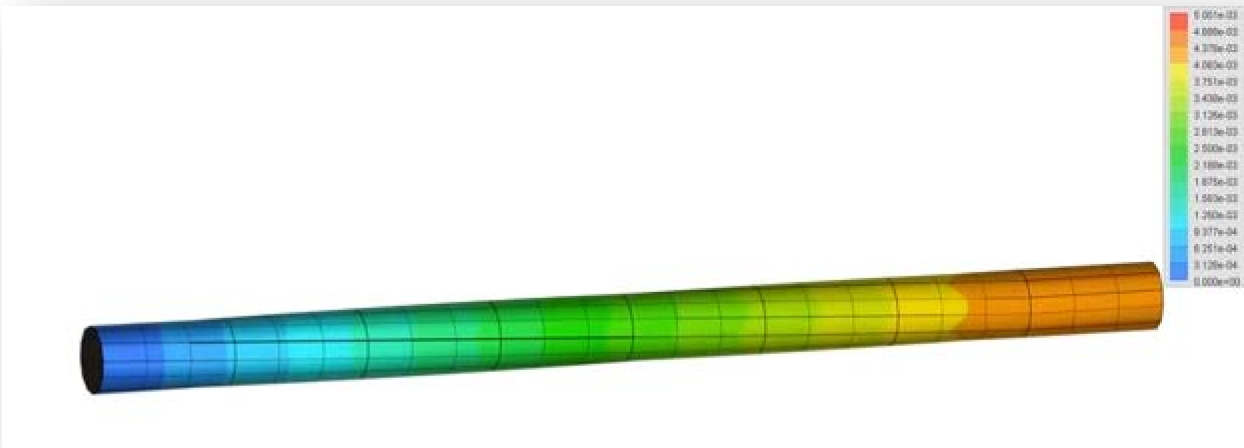


A video for Torsion (circular) test
http://www.youtube.com/watch?v=76dVeHSWDZk

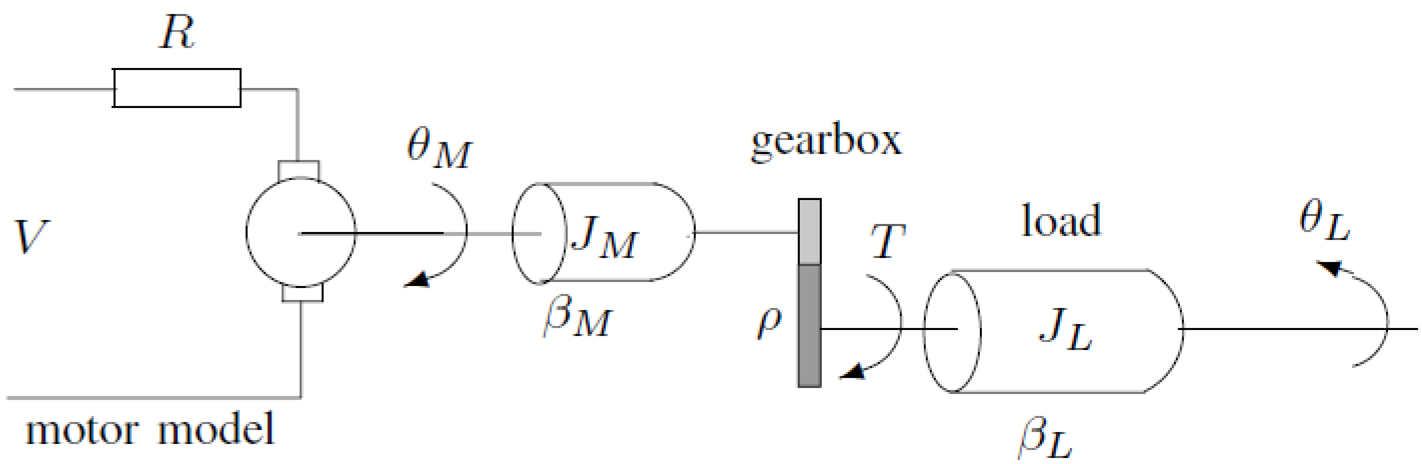


Figure: The benchmark position servomechanism model

The plant model

$$\dot{\omega}_L = -\frac{k_\theta}{J_L}\left(\theta_L - \frac{\theta_M}{\rho}\right) - \frac{\beta_L}{J_L}\omega_L$$

$$\dot{\omega}_M = \frac{k_T}{J_M}\left(\frac{V - k_T\omega_M}{R}\right) - \frac{\beta_M\omega_M}{J_M} + \frac{k_\theta}{\rho J_M}\left(\theta_L - \frac{\theta_M}{\rho}\right)$$

The plant model

$$\dot{x}_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_\theta}{J_L} & -\frac{\beta_L}{J_L} & \frac{k_\theta}{\rho J_L} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_\theta}{\rho J_M} & 0 & -\frac{k_\theta}{\rho^2 J_M} & -\frac{\beta_M + k_T^2/R}{J_M} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_T}{R J_M} \end{bmatrix} V.$$

$$\theta_L = [1 \quad 0 \quad 0 \quad 0] x_p,$$

$$T = \left[k_\theta \quad 0 - \frac{k_\theta}{\rho} \quad 0 \right] x_p.$$

An SISO system
The input: V
The Feedback: θ_L

Table: The system parameters of the servomechanism

Symbol	Value (SI units)	Symbol	Value (SI units)
k_θ	1280.2	ρ	20
k_T	10	β_M	0.1
J_M	0.5	β_L	25
J_L	$50 J_M$	R	20

A proposed approximate realization method of fractional derivative

$$s^\alpha \approx \left(\frac{1 + \frac{s}{\frac{d}{b}\omega_b}}{1 + \frac{s}{\frac{b}{d}\omega_h}} \right)^\alpha$$

$$s^\alpha \approx K \left(\frac{ds^2 + bs\omega_h}{d(1-\alpha)s^2 + bs\omega_h + d\alpha} \right) \prod_{k=-N}^{N} \frac{s + \omega'_k}{s + \omega_k},$$

Better than Oustaloup's fractional operator approximation method

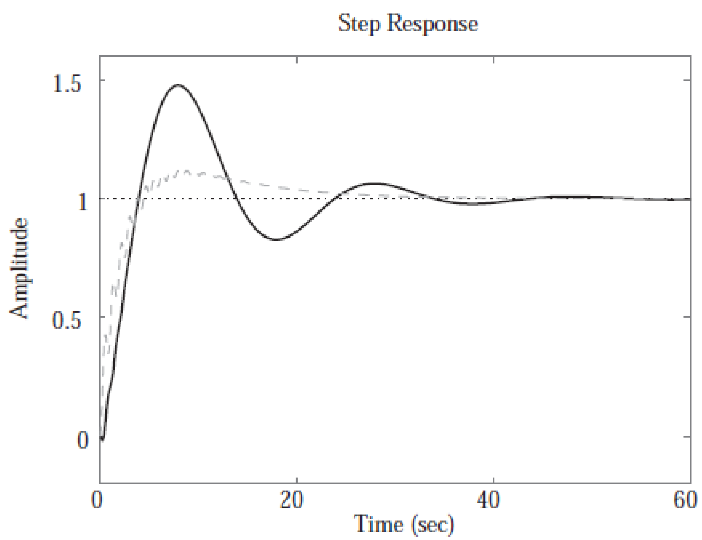
The IOPID

Using ITAE

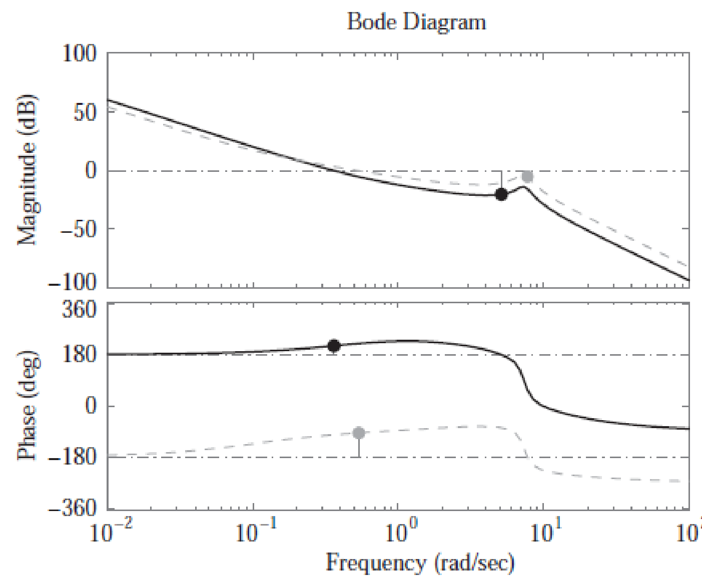
$$G_{c1} = 41.94 + \frac{21.13}{s} - 8.26s$$

Using ISE

$$G_{c1} = 110.09 + \frac{10.65}{s} - 30.97s$$



(a) Step responses



(b) Bode plots

Figure: the best FOPID Controllers. Solid line: ITAE; Broken line: ISE

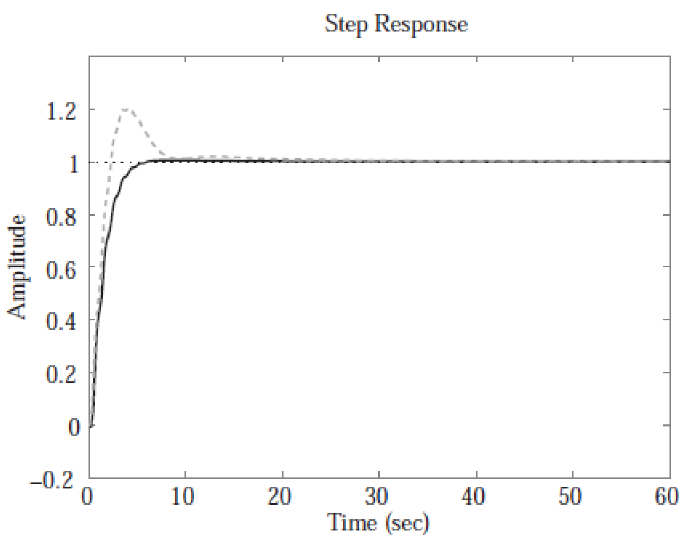
The FOPID with pre fixed $\lambda = 0.5$ and $\mu = 0.6$

Using ITAE

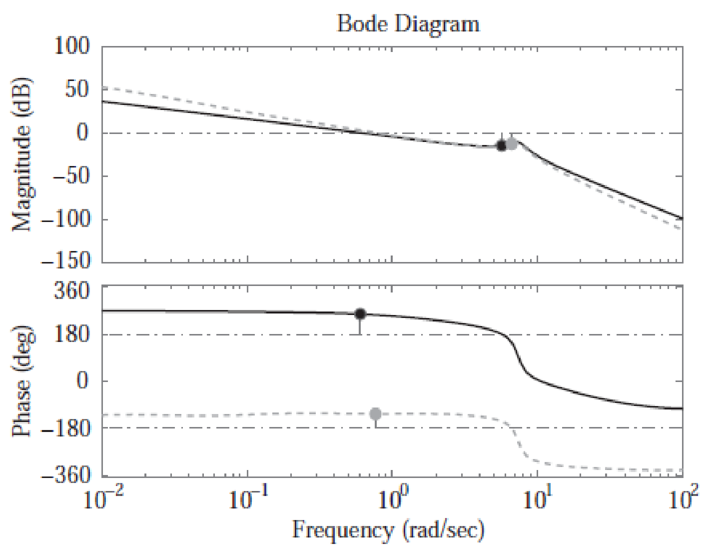
$$G_{c3} = 135.12 + \frac{0.01}{s^{0.5}} - 31.6s^{0.6}$$

Using ISE

$$G_{c4} = 61.57 + \frac{91.95}{s^{0.5}} - 2.33s^{0.6}$$



(a) Step responses

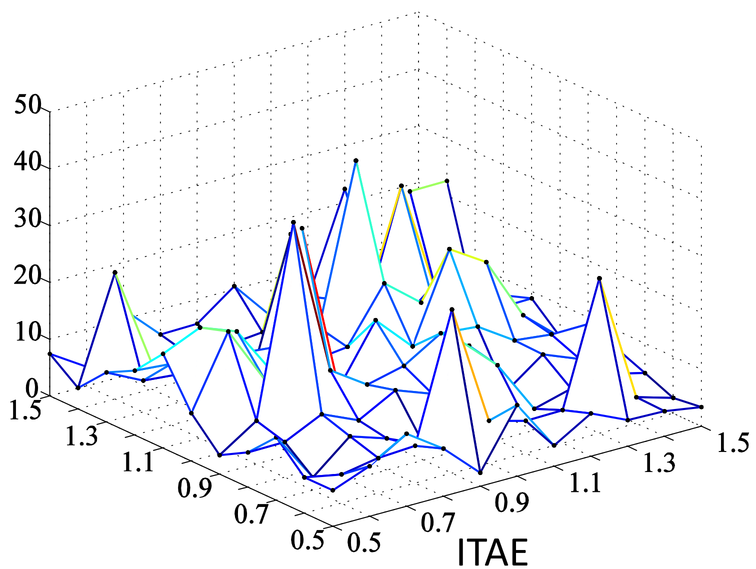


(b) Bode plots

Figure: the best IOPID Controllers. Solid line: ITAE; Broken line: ISE

Searching for the best fractional orders λ and μ

N = 2, 4, 6, ...



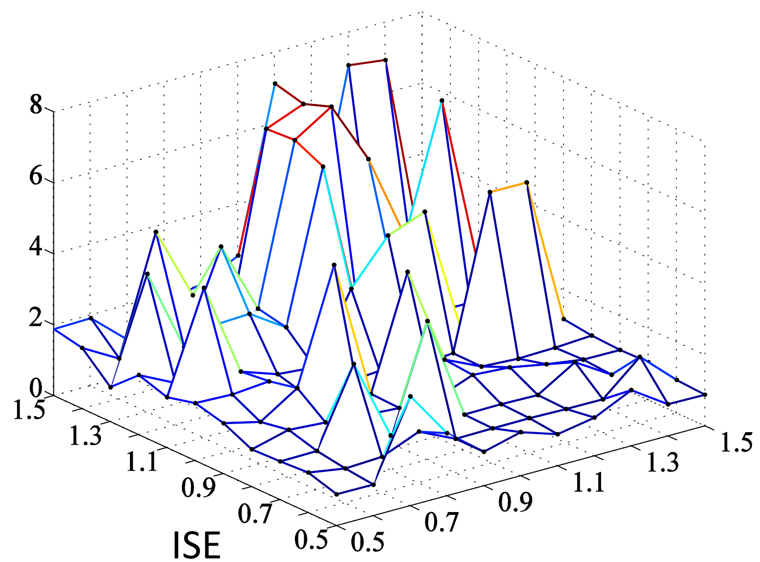
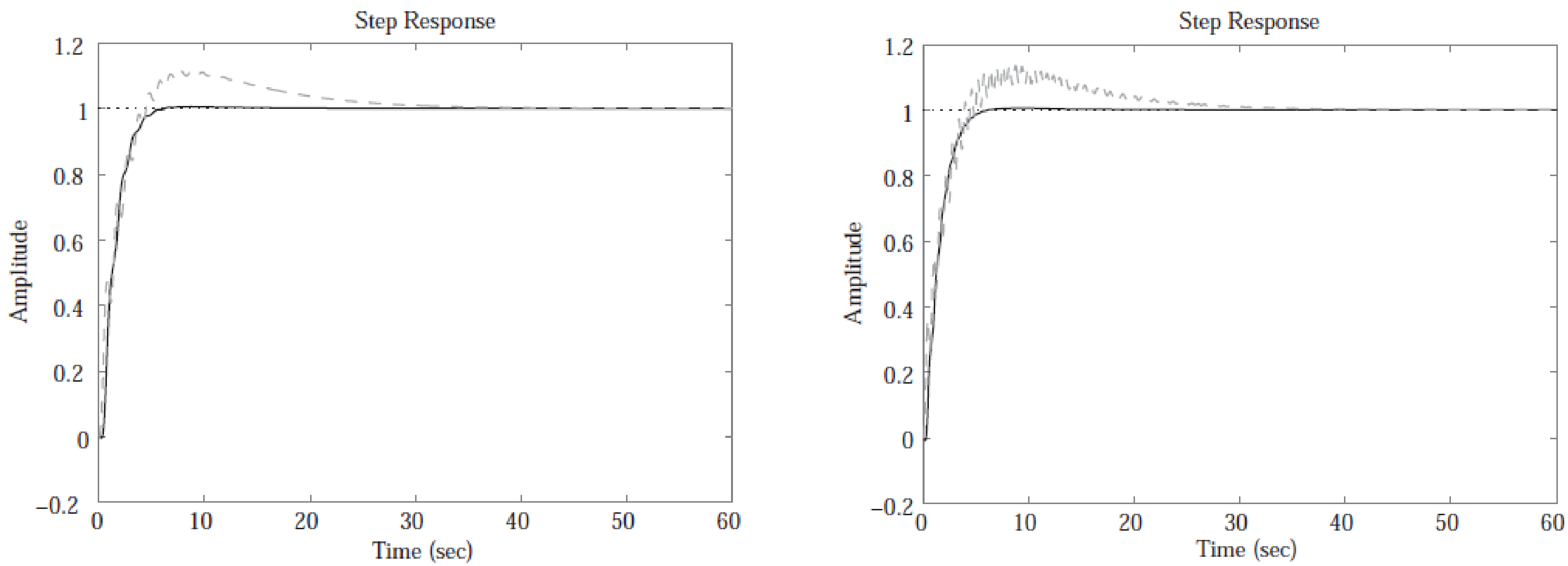


Figure: Searching for the best fractional orders (N = 4)

Robustness to load change

±50% load variation



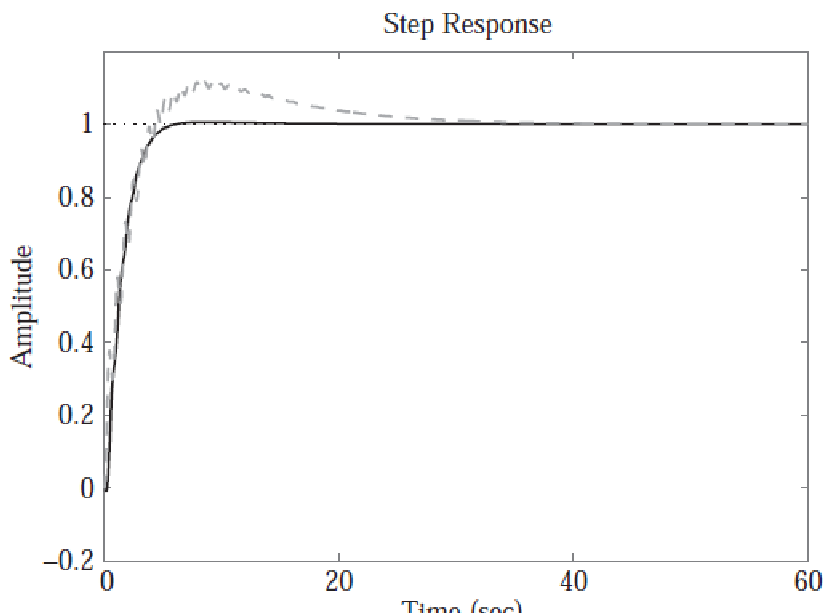
(a) +50% load variation

(b) −50% load variation

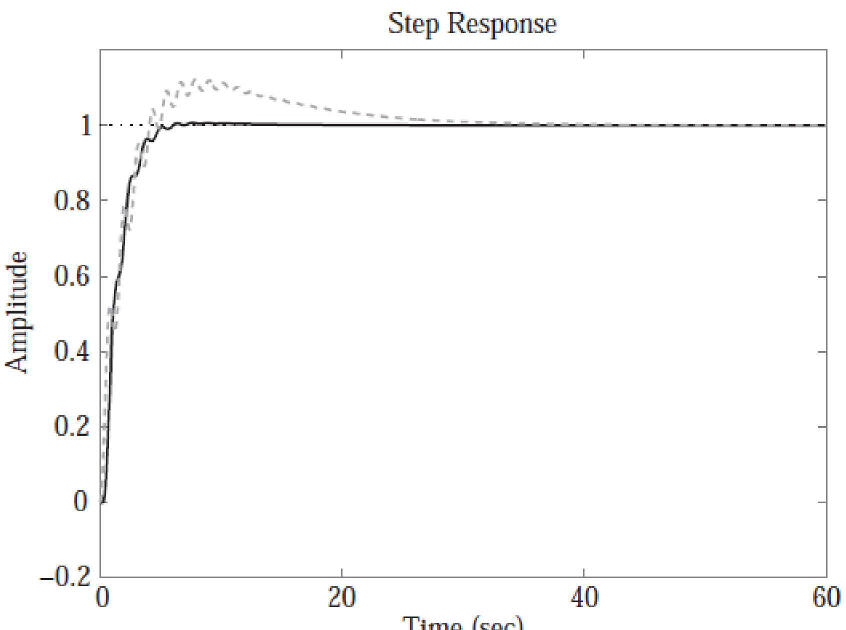
Figure: Step responses comparison with N = 4 (Dotted: Best PID; Solid line: Best FOPID)

Robustness to Elasticity parameter change

$\pm 50\%\ k_\theta$ load variation



(a) best PID



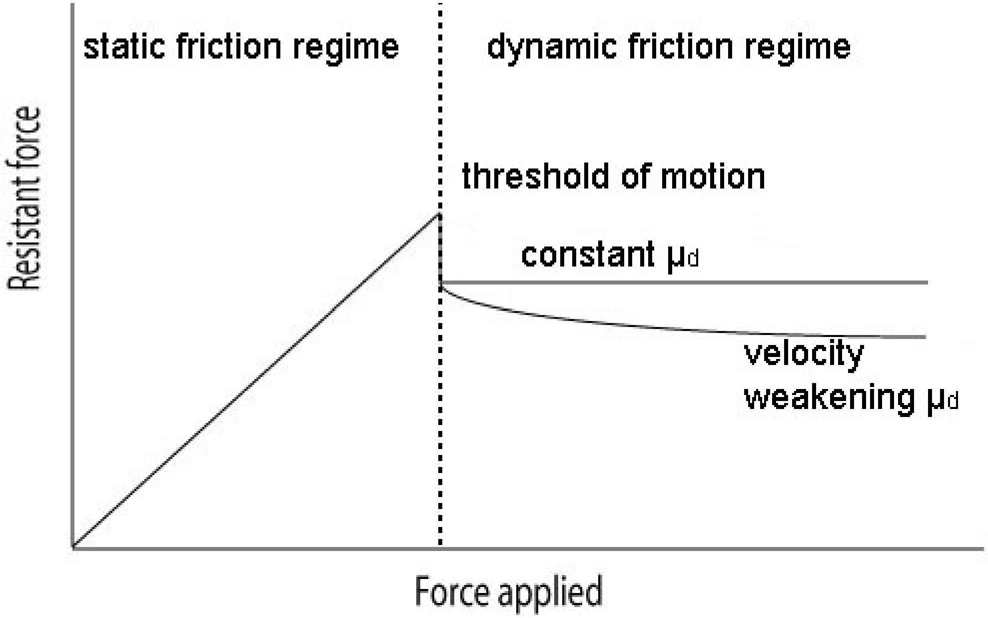
(a) best PID

Figure: Responses comparison when k_θ increases 50% Dashed line: Best IO Controllers; Solid line: Best FO Controllers

Scope of the book

The impact of friction force

Tracking error
Limit cycle
Spiky performance
Etc.



Recall the plant model and the controller in Chapter 6

Second order position servo

$$P(s) = \frac{1.52}{s(0.4s + 1)}$$

Friction was not considered back to ch6

Now, take it into consideration below

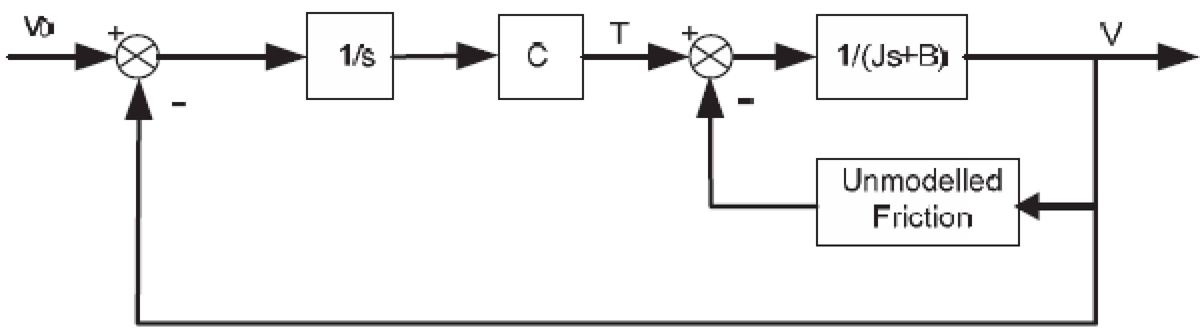
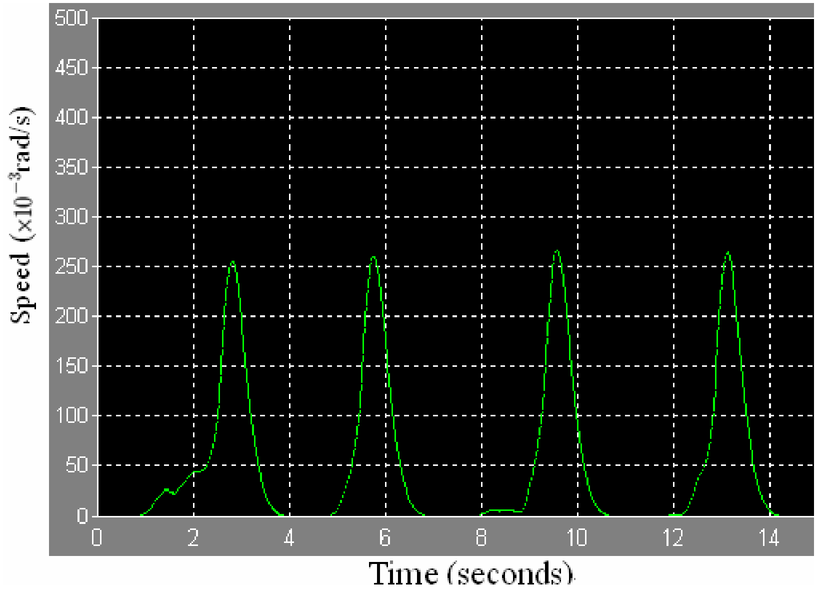


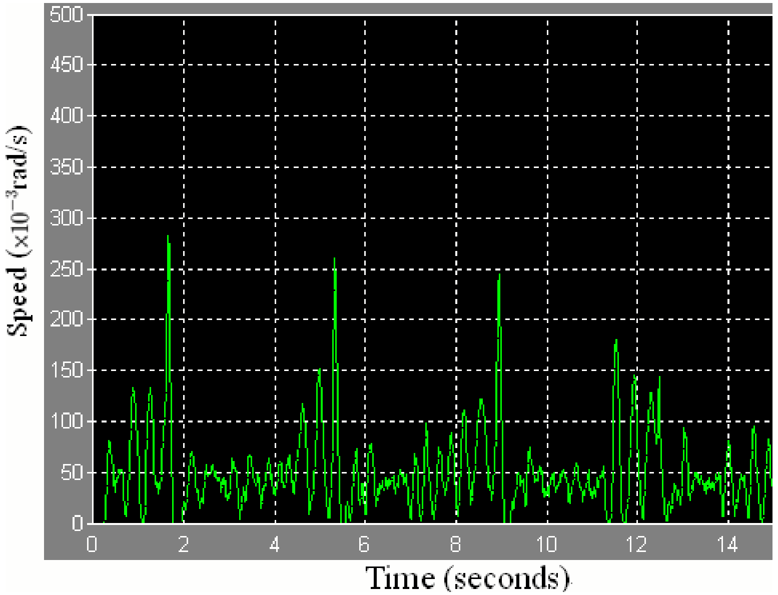
Figure: Position tracking control equivalent diagram with constant speed reference and friction

Notice

Ultra-low speed !



IOPI: C(s)=2.56(1+1.166/s)



FOPD: C(s)=13.86(1+0.368/s)

Figure: Speed outputs of position tracking with constant speed reference using optimized IOPI and designed FOPD

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 40, NO. 3, MARCH 1995 419

A New Model for Control of Systems with Friction

C. Canudas de Wit, *Associate, IEEE*, H. Olsson, *Student Member, IEEE*, K. J. Åström, *Fellow, IEEE*, and P. Lischinsky

$$
\begin{aligned}
\dot{z} &= v - \frac{|v|}{g(v)} z, \\
F &= \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v, \\
\sigma_0 g(v) &= F_C + (F_S - F_C) e^{-(v/v_s)^2},
\end{aligned}
$$

Figure: Parameter values in the LuGre model

σ_0	10^5	[N/m]
σ_1	$\sqrt{10^5}$	[Ns/m]
σ_2	0.4	[Ns/m]
F_C	1	[N]
F_S	1.5	[N]
v_S	0.001	[m/s]

where the average deflection of the bristles is denoted by z; v is the relative velocity between the two surfaces; the function g is positive and depends on many factors such as material properties, lubrication and temperature; σ_0 is the stiffness, and σ_1, σ_2 are damping coefficients; F_C is the Coulomb friction level; F_S is the level of the stiction force; and v_s is the Stribeck velocity. The values of those parameters in (18.1)

The describing function method

The ratio of the first harmonic of output to input

$$y_1(t) = a\cos\omega t + b\sin(\omega t) = c\sin(\omega t + \varphi),$$

where

$$a = \frac{2}{\pi}\int_0^{\pi} y(t)\cos(\omega t)\mathrm{d}(\omega t), \quad b = \frac{2}{\pi}\int_0^{\pi} y(t)\sin(\omega t)\mathrm{d}(\omega t),$$
$$c = \sqrt{a^2 + b^2}, \quad \varphi = \arctan(a/b).$$

Decoupling the linear and nonlinear part

$$N(T_0, A, \omega) = [|N|e^{\phi}, v_a]$$

T_0 is the mean torque

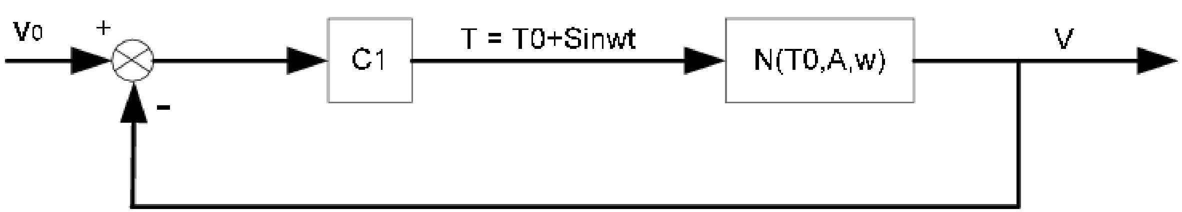
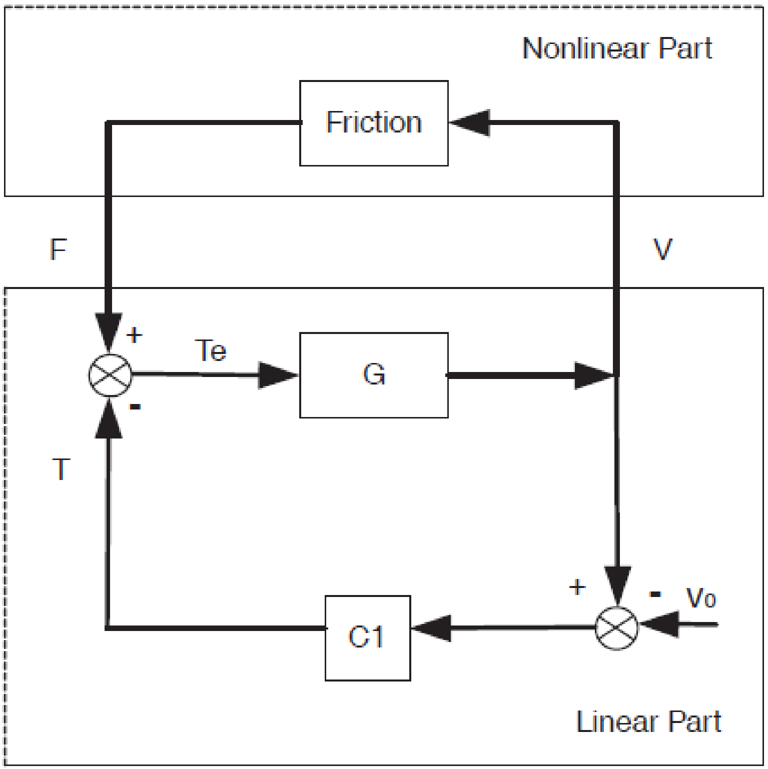
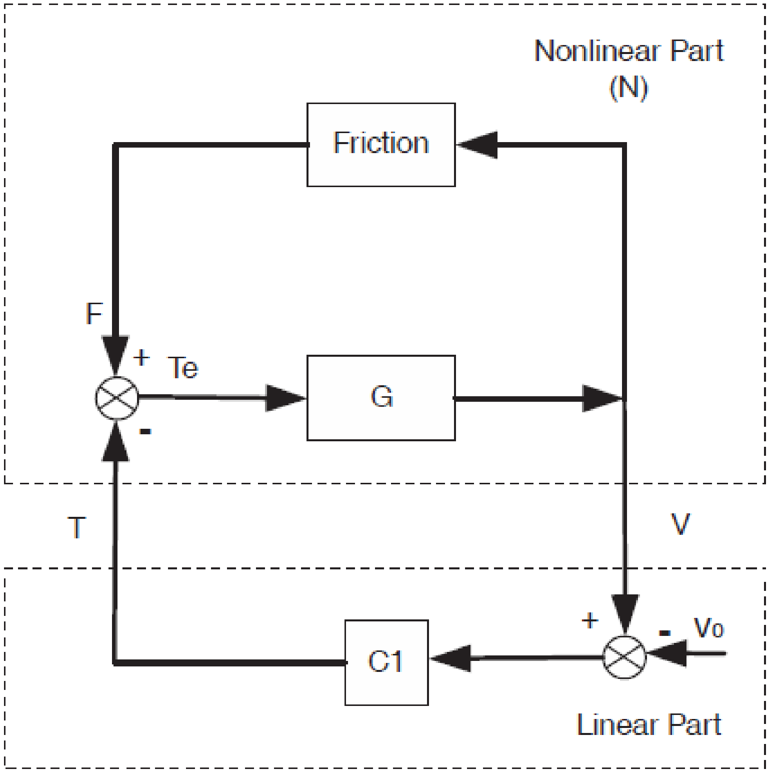


Figure: Approximation closed-loop system with linear and nonlinear parts



(a) Nonlinear part with only friction



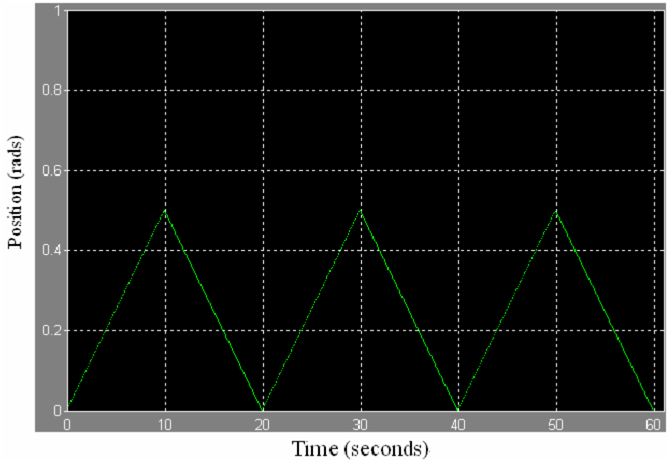
(b) Nonlinear part with friction and system dynamics

Figure: Two methods of uncoupling linear and nonlinear parts

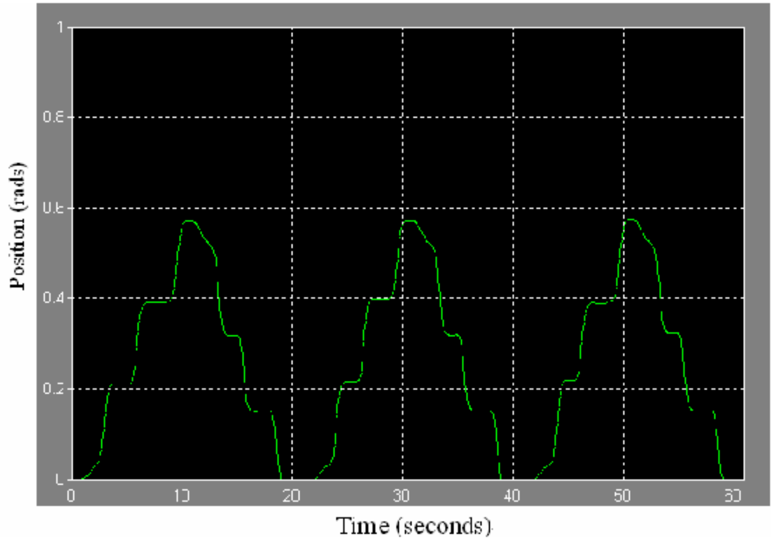
Ultra-low speed reference

FOPD: C(s)=2.56(1+1.166/s)

IOPI: C(s)=13.86(1+0.368/s)



Varying low speed (±0.05rad/s) position reference for tracking



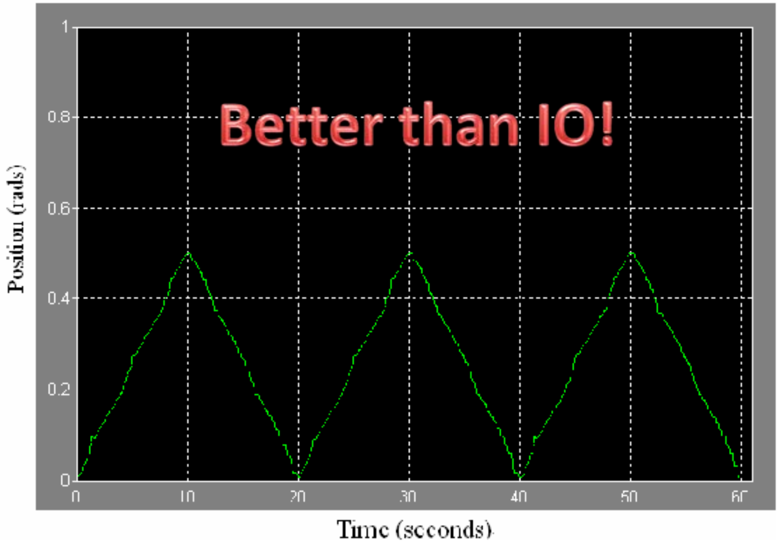


Figure: Position tracking outputs with varying low speed reference using IOPI/FOPD

Scope of the book

The integrator: a double-edged sword

Eliminating the steady state error
Introducing a 90° phase lag

Attempts of improvement

CCI: Clegg conditional integrator
ICI: intelligent conditional integrator
MICI: modified intelligent conditional integrator
FOCI: fractional order conditional integrator
OFOCI: optimized fractional order conditional integrator

A Nonlinear Integrator for Servomechanisms

J. C. CLEGG
ASSOCIATE MEMBER AIEE

MARCH 1958

CCI

《Information and Control》 1990-06 Add to Favorite Get Latest Update

A NEW INTELLIGENT INTEGRATOR

WANG Yong CAO Zhengping XIANG Guobo (Dept of Automation,Wuhan University of Technology)

ICI

The input and output of the CCI

For input $x(t)Asin(wt)$

$$y_C(\varphi) = \int_0^{\varphi/\omega} x(t)\mathrm{d}t = \frac{1}{\omega}\int_0^{\varphi} A\sin(\omega t)\mathrm{d}(\omega t)$$
$$= \frac{A}{\omega}(1-\cos(\varphi)).$$

The describing function of the CCI

$$N_C(A,\omega) = \frac{2j}{\pi A}\int_0^{\pi}\frac{A}{\omega}(1-\cos\varphi)e^{-j\varphi}\mathrm{d}\varphi = \frac{4}{\pi\omega}\left(1 - j\frac{\pi}{4}\right),$$
$$|N_C(A,\omega)| = \frac{1}{\omega}\sqrt{1+\left(\frac{4}{\pi}\right)^2} = \frac{1.62}{\omega},$$
$$\arg N_C(A,\omega) = -\arctan\frac{\pi}{4} \simeq 0.212\pi(rad.) = 38.15^\circ.$$

The input and output of the FOCI

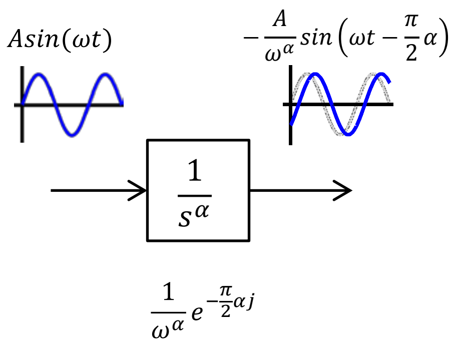
The FO integrator

$$_{-\infty}D_t^{-\alpha} A\sin(\omega t) = \frac{A}{\omega^\alpha}\sin\left(\omega t - \frac{\alpha\pi}{2}\right),$$

The output of the FOCI

$$y_F(t) = \begin{cases} {}_0D_t^\alpha u(t) = \frac{A}{\omega^\alpha}[\sin(\omega t - \frac{\alpha\pi}{2}) + \sin(\frac{\alpha\pi}{2})], 0 \leqslant \omega t < \frac{\pi}{2}; \\ k\frac{A}{\omega}, \frac{\pi}{2} \leqslant \omega t < \pi; \\ 0, \omega t = \pi; \\ {}_\pi D_t^\alpha u(t) = \frac{A}{\omega^\alpha}[\sin(\omega t - \frac{\alpha\pi}{2}) - \sin(\frac{\alpha\pi}{2})], \pi \leqslant \omega t < \frac{3\pi}{2}; \\ -k\frac{A}{\omega}, \frac{3\pi}{2} \leqslant \omega t < 2\pi; \\ 0, \omega t = 2\pi; \end{cases}$$

k is the “holding” parameter



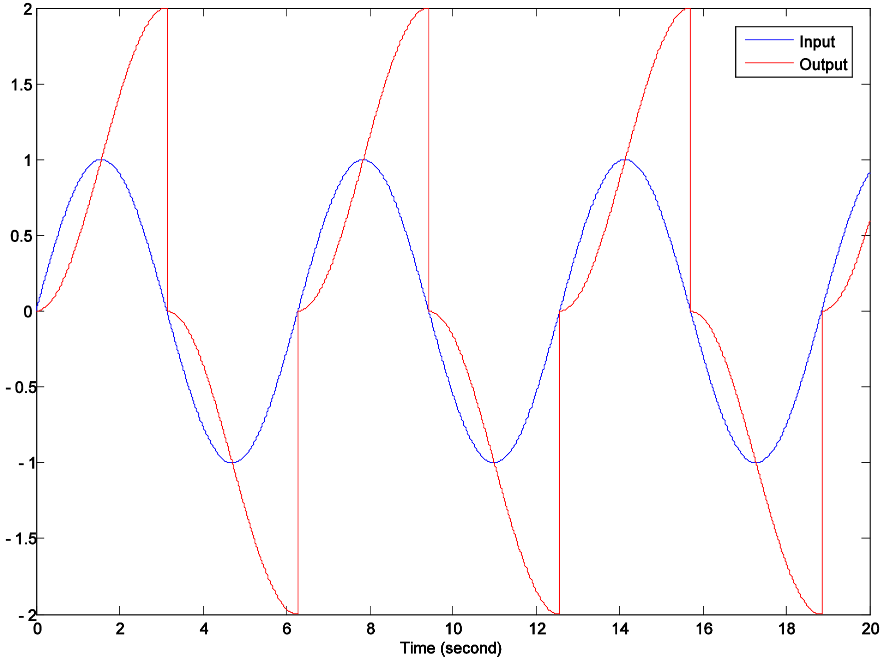


Figure: Input and output signals of CCI

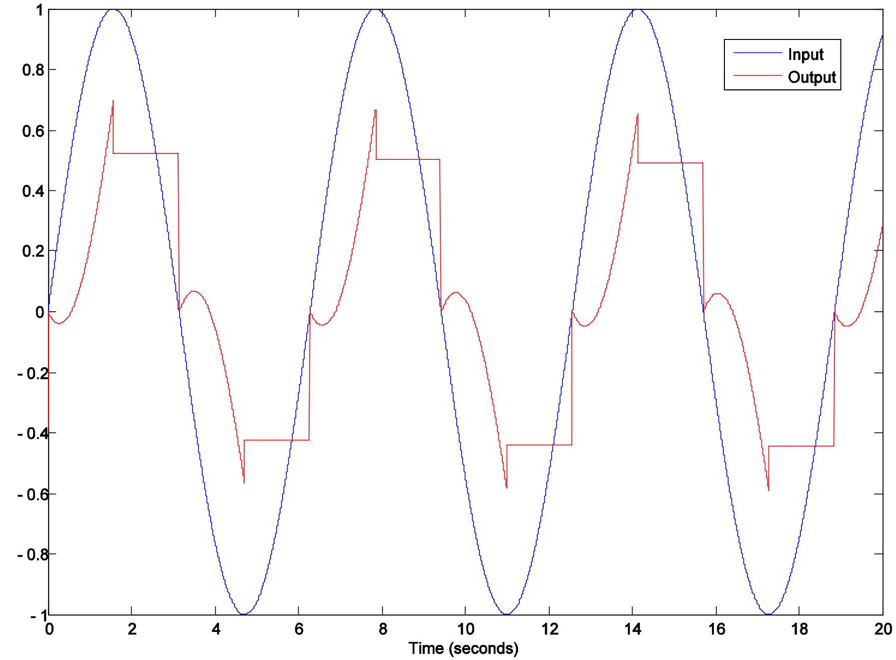


Figure: Input and output signals of FOCI

Optimality criteria

- Min $\frac{\partial J_{xF}(\alpha,k)}{\partial \alpha} = 0$

  $J_{xF}(\alpha, k) = \frac{C_{xF}}{C_{1F}}$ is the cost function, C_{xF} is the x-th order harmonic

- Phase delay

  $\varphi_{1F} = \arctan\left(\frac{a_{1F}}{b_{1F}}\right) = \varphi^*$

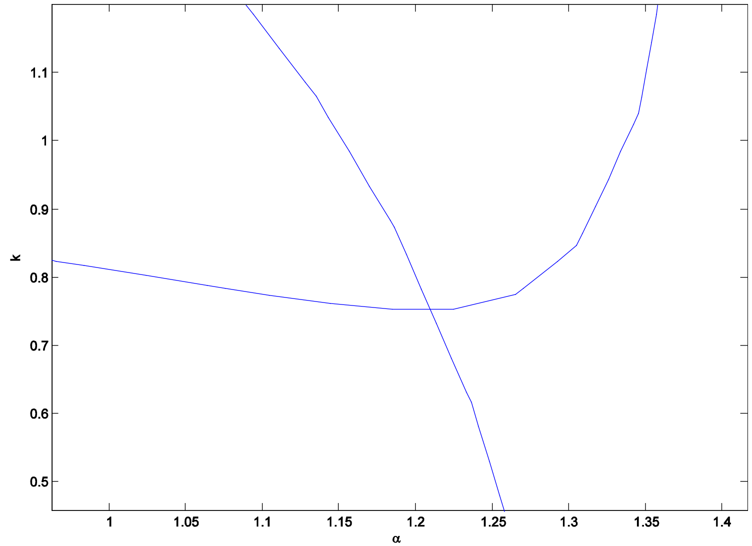


Figure: Optimized solutions with phase delays $-32°$ according to the optimality criteria

Table: Performance comparison with the third-order harmonic component ratio minimum specification

Type	$N(j\omega)$	φ_1	$\frac{c_3}{c_1}$ (%)	$\frac{c_5}{c_1}$ (%)	$\frac{c_7}{c_1}$ (%)	$\frac{c_9}{c_1}$ (%)	$\sum$ (%)
CCI	$\frac{1.62}{\omega}e^{-j0.212\pi}$	$-38.1°$	26.2	15.8	11.2	8.7	61.9
ICI	$\frac{1.08}{\omega}e^{-j0.153\pi}$	$-27.6°$	9.84	13.78	7.03	7.22	37.87
$OFOCI_1$	$\frac{0.42}{\omega^{1.273}}e^{-j0.212\pi}$	$-38.1°$	6.23	17.35	5.24	8.51	37.33
$OFOCI_2$	$\frac{0.52}{\omega^{1.209}}e^{-j0.178\pi}$	$-32.0°$	0.31	16.29	5.77	8.14	30.51
$OFOCI_3$	$\frac{0.67}{\omega^{1.126}}e^{-j0.153\pi}$	$-27.6°$	4.38	15.04	6.25	7.65	33.32
$OFOCI_4$	$\frac{1.33}{\omega^{0.530}}e^{-j0.122\pi}$	$-22.0°$	12.48	12.73	7.10	6.75	39.06

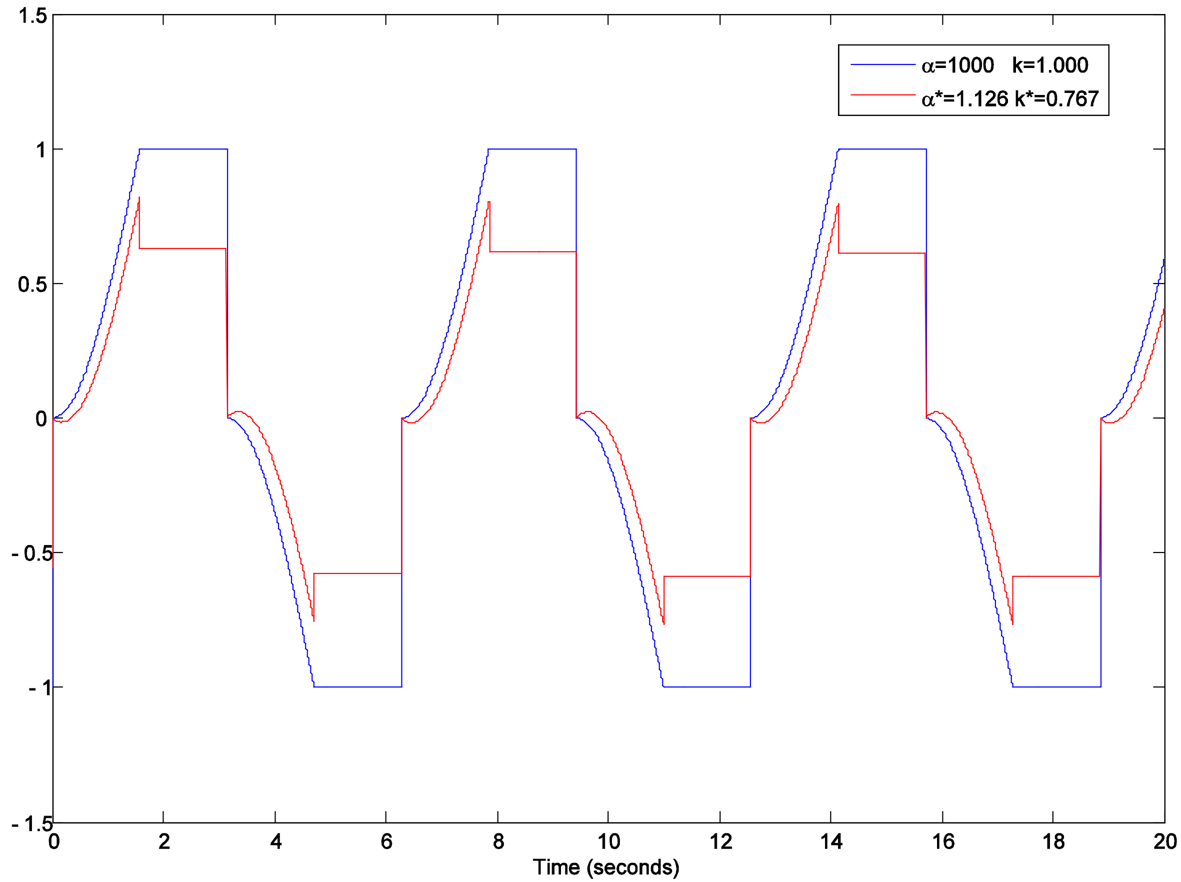


Figure: Output comparison of the ICI and the OFOCI3

This is the end of session VI

Questions?