Turkish National Meeting on Automatic Control
(TOK 2013) , Sept. 25, 2013, Malatya, Turkey

A Tutorial on Fractional Order Motion Control

Part III: Fractional Order Position Servo

Prof. YangQuan Chen
Zhuo Li, PhD student
EECS, University of California, Merced
(ychen53, zli32)@ucmerced.edu
MESA Lab http://mechatronics.ucmerced.edu

- Fractional Order Motion Controls

John Wiley & Sons, Inc.

Hardcover, 454 pages, December 2012

- Dr. Ying Luo

Automation Science and Engineering, South China University of Technology, Guangzhou, PR China

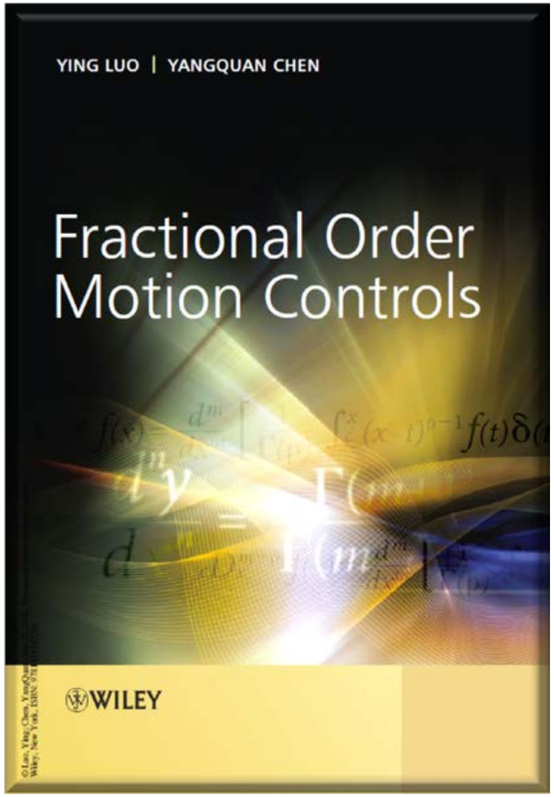
- Dr. YangQuan Chen

Director, Mechatronics, Embedded Systems and Automation (MESA) LAB

MEAM/EECS, School of Engineering,

University of California, Merced

Email: yqchen@ieee.org



Scope of the book

Scope of the book

The model to be discussed

Second order position servo

$$P(s) = \frac{1}{s(Ts+1)}$$

Gain and phase in frequency domain

$$\angle P(j\omega) = -tan^{-1}(\omega T) - \frac{\pi}{2}$$

$$|P(j\omega)| = \frac{1}{\omega\sqrt{1+(\omega T)^2}}$$

The traditional integer order PD controller

$$C(s) = K_p(1 + K_d s)$$

Integer order PD controller design using “flat phase” concept

$$G(s) = P(s)C(s) = \frac{K_p(1 + K_d s)}{s(Ts + 1)}$$

$$\angle G(j\omega) = \arctan(\omega K_d) - \arctan(\omega T)$$

$$\left.\frac{d\angle G(j\omega)}{d\omega}\right|_{\omega=\omega_c} = \frac{K_d}{1 + (K_d\omega_c)^2} - \frac{T}{1 + (T\omega_c)^2}$$

$$\Rightarrow \qquad K_d = \frac{1}{T{\omega_c}^2}$$

This means: given a ω_c, the phase margin at the flat part is fixed. No way to adjust !!

The fractional order proportional and derivative controller PD^{μ}

$$C(s) = K_p(1 + K_d s^{\mu})$$

Gain and phase in frequency domain

$$\angle C(j\omega) = tan^{-1}\frac{sin\frac{(1-\mu)\pi}{2} + K_d\omega^{\mu}}{cos\frac{(1-\mu)\pi}{2}} - \frac{(1-\mu)\pi}{2}$$

$$|C(j\omega)| = K_p\sqrt{\left(1 + K_d\omega^{\mu}cos\frac{\mu\pi}{2}\right)^2 + (K_d\omega^{\mu}sin\frac{\mu\pi}{2})^2}$$

Formulas for determining the parameters of FO PD controller

$$\left\{\begin{aligned}
K_d &= \frac{1}{\omega_c^{\mu}}\tan[\phi+\tan^{-1}(\omega_c T)-\frac{\mu\pi}{2}+\pi]\cos\frac{(1-\mu)\pi}{2} \\
&\quad -\frac{1}{\omega_c^{\mu}}\sin\frac{(1-\mu)\pi}{2}. \qquad (7.1)\\
K_d &= \frac{-B\pm\sqrt{B^2-4A^2\omega_c^{2\mu}}}{2A\omega_c^{2\mu}}, \qquad (7.2)\\
A &= \frac{T}{1+(\omega_c T)^2},\\
B &= 2A\omega_c^{\mu}\sin\frac{(1-\mu)\pi}{2}-\mu\omega_c^{\mu-1}\cos\frac{(1-\mu)\pi}{2}.\\
& |G(j\omega_c)| \\
&= |C(j\omega_c)||P(j\omega_c)| \qquad (7.2)\\
&= \frac{K_p\sqrt{(1+K_d\omega_c^{\mu}\cos\frac{\mu\pi}{2})^2+(K_d\omega_c^{\mu}\sin\frac{\mu\pi}{2})^2}}{\omega_c\sqrt{1+(\omega_c T)^2}}\\
&= 1.
\end{aligned}\right.$$

The procedure to obtain the controller parameters

(1) Given ω_c, the gain crossover frequency.
(2) Given Φ_m, the desired phase margin.
(3) Plot the curve 1, K_d with respect to μ, according to (6.11).
(4) Plot the curve 2, K_d with respect to μ, according to (6.14).
(5) Obtain the μ and K_d from the intersection point on the above two curves.
(6) Calculate the K_p from (6.15).

Performance comparison

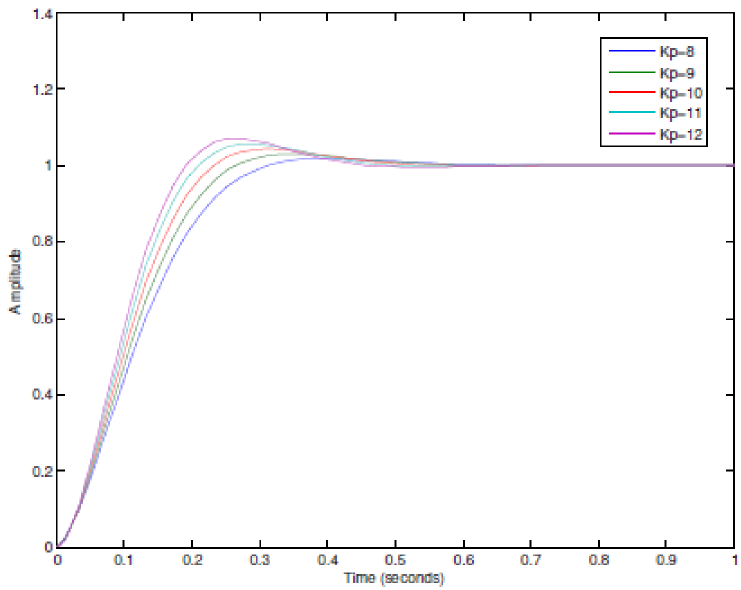


Figure: Step responses with the ITAE optimum proportional controller

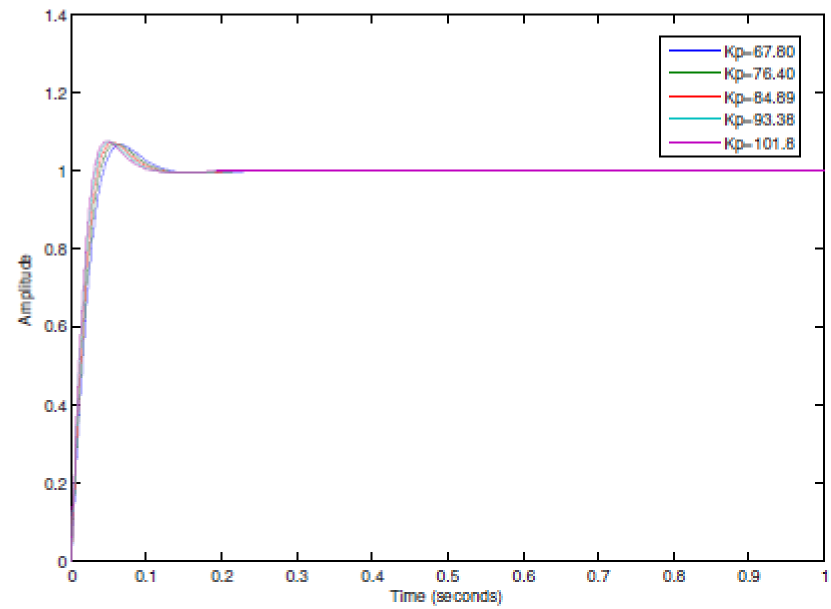
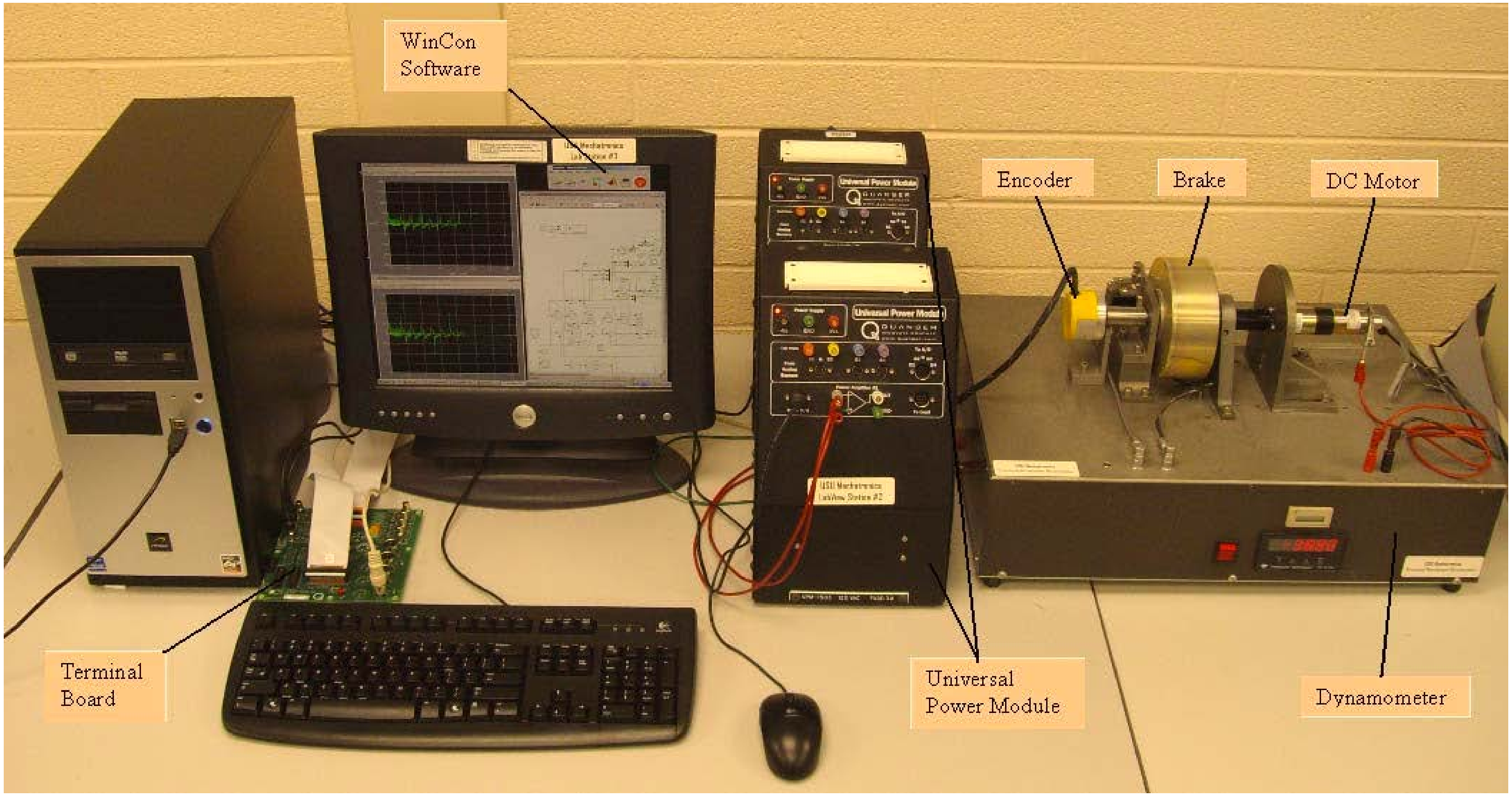


Figure: Step responses with PD^{λ} controller.

The experimental platform

A general purpose fractional horsepower dynamometer
Quanser DAQ
Matlab/Simulink



The model of the platform

Second order position servo

$$P(s) = \frac{1.52}{s(0.4s+1)}$$

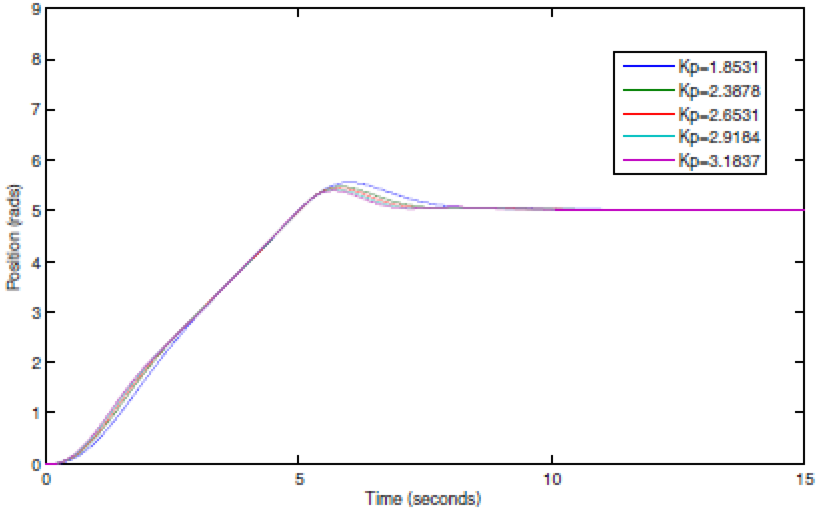


Figure: Step position responses with the ITAE optimal proportional controller

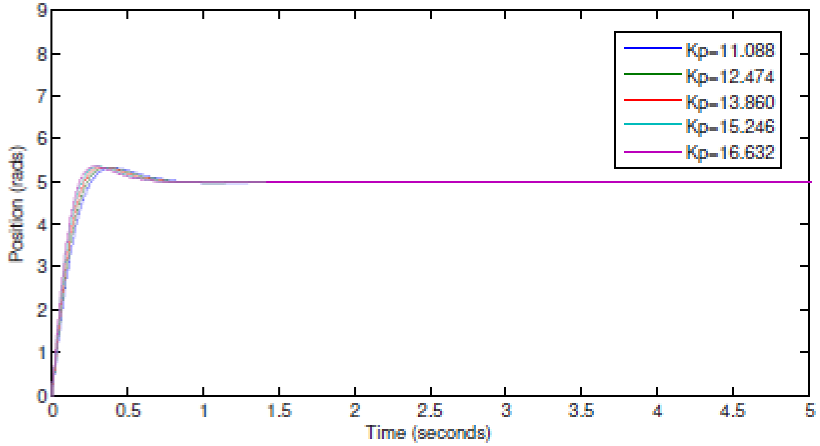


Figure: Step responses with PD^{λ} controller.

- Some of the methods to implement s^{λ}

- The CRONE toolbox - Alain Oustaloup

From fractal robustness to the CRONE control

Alain Oustaloup, Jocelyn Sabatier and Patrick Lanusse
CRONE Team - LAP - Université Bordeaux I - ENSERB
351, cours de la Libération F-33405 Talence Cedex - FRANCE
email : oustaloup@lap.u-bordeaux.fr

December 4, 2000

- Impulse response invariant discretization (IRID) – Yangquan Chen

MATLAB CENTRAL

Search: File Exc

File Exchange | Answers | Newsgroup | Link Exchange | Blogs | Trendy | Cody | Contest | MathWorks.com

File Exchange

Impulse response invariant discretization of fractional order integrators/differentiators

by YangQuan Chen
05 Sep 2008 (Updated 05 Sep 2008)

compute a discrete-time finite dimensional (z) transfer function to approximate s^r, r is real numbe

Be the first to rate this file!
8 Downloads (last 30 days)
File Size: 3.3 KB
File ID: #21342

The model to be discussed

Second order position servo

$$P(s) = \frac{1}{s(Ts+1)}$$

The fractional order [PD] controller $(PD)^{\mu}$

$$C(s) = K_p(1 + K_d s)^{\mu}$$

Gain and phase in frequency domain

$$\angle C(j\omega) = \mu tan^{-1}(\omega K_d)$$
$$|C(j\omega)| = K_p[1 + (K_d\omega)^2]^{\frac{\mu}{2}}$$

Some basic techniques for dealing with complex numbers

$$z = x + jy = |z|e^{j\varphi}$$
$$z^\alpha = (x + jy)^\alpha = (|z|e^{j\varphi})^\alpha = |z|^\alpha e^{j\varphi\alpha}$$
$$|z| = \sqrt{x^2 + y^2}$$
$$|z^\alpha| = |z|^\alpha = (\sqrt{x^2 + y^2})^\alpha$$
$$\angle z^\alpha = \varphi\alpha$$

The "flat phase" tuning equations

$$\left\{\begin{aligned} |G_3(j\omega_c)| &= |C_3(j\omega_c)||P(j\omega_c)| \\ &= K_{p3}\frac{(1+(K_{d3}\omega_c)^2)^{\frac{\mu}{2}}}{\sqrt{(T\omega_c^2)^2+\omega_c^2}} \\ &= 1. \\ \left.\frac{\mathrm{d}(\angle(G_3(j\omega)))}{\mathrm{d}\omega}\right|_{\omega=\omega_c} &= \frac{\mu K_{d3}}{1+(K_{d3}\omega_c)^2} - \frac{T}{1+(T\omega_c)^2} \\ &= 0, \\ \angle[G_3(j\omega)]|_{\omega=\omega_c} &= \mu\tan^{-1}(\omega_c K_{d3}) - \tan^{-1}(\omega_c T) - \frac{\pi}{2} \\ &= -\pi + \phi_m. \end{aligned}\right.$$

The procedure to obtain the controller parameters

(1) Given ω_c, the gain crossover frequency.
(2) Given Φ_m, the desired phase margin.
(3) Plot the curve 1, K_{d3} with respect to μ, according to (7.13).
(4) Plot the curve 2, K_{d3} with respect to μ, according to (7.16).
(5) Obtain the K_{d3} and μ from the intersection point on the above two curves.
(6) Calculate the K_{p3} from (7.17).

Implementation of FO [PD]

- The FO operator $(1+\tau s)^{\mu}$ for the FO [PD] controller can be implemented by modifying the code of the IRID.
- A discrete-time finite dimensional z transfer function is computed to approximate a continuous-time fractional order low-pass filter $\frac{1}{(\tau s+1)^{\mu}}$
- Implement $\frac{1}{(\tau s+1)^{\mu}}$, $\mu \in (0,1)$ first
- Then change $\mu \in (-1,0)$

- Plant model

$$P(s) = \frac{1}{s(0.4s+1)}$$

- Design specifications

$\omega_c = 10\, rad/s$
$\Phi_m = 70°$
Flat phase at ω_c

- IOPID parameters

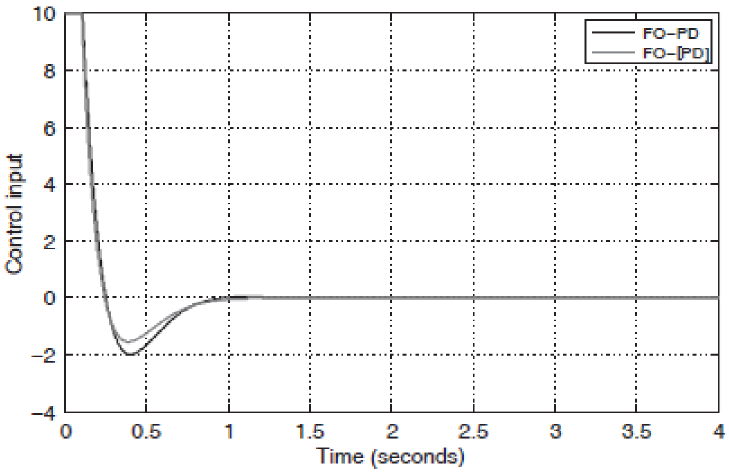
$Kp = 23.078, Kd = 0.102, Ki = -4.625$
Unstable

- FO PD

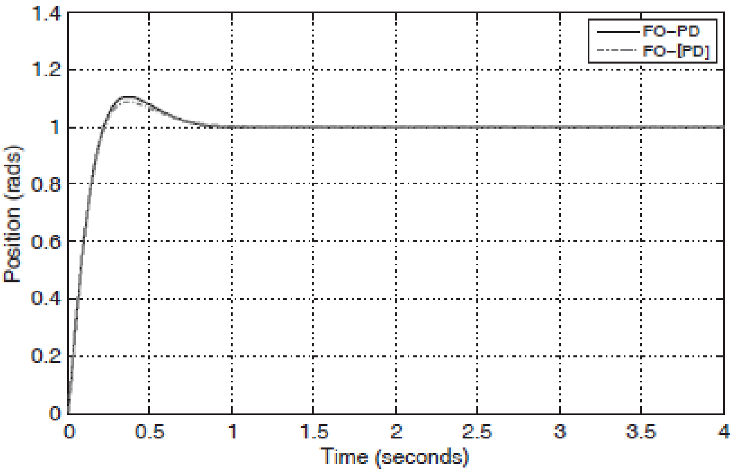
$Kp = 16.784, Kd = 0.368, \lambda = 0.835$

- FO [PD]

$Kp = 13.860, Kd = 0.299, \lambda = 0.783$



Simulation. Control input signals with two FO controllers (T = 0.4s)



Simulation. Step responses comparison with two FO controllers (T = 0.4s)

Simulation example 2

- Plant model

$$P(s) = \frac{1}{s(0.04s+1)}$$

- Design specifications

$\omega_c = 10\, rad/s$
$\Phi_m = 70°$
Flat phase at ω_c

- IOPID parameters

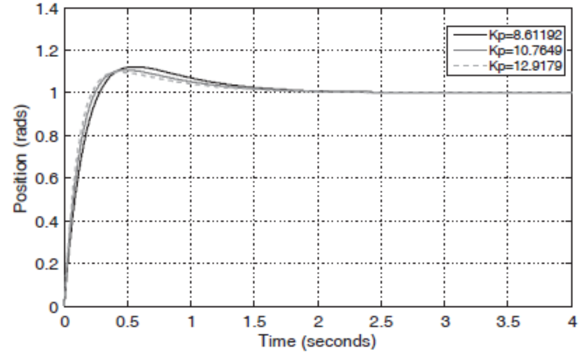
$Kp = 10.76, Kd = 0.018, Ki = 1.567$

- FO PD

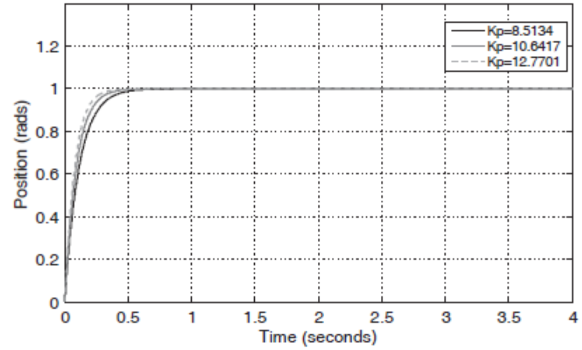
$Kp = 10.64, Kd = 0.005, \lambda = 0.779$

- FO [PD]

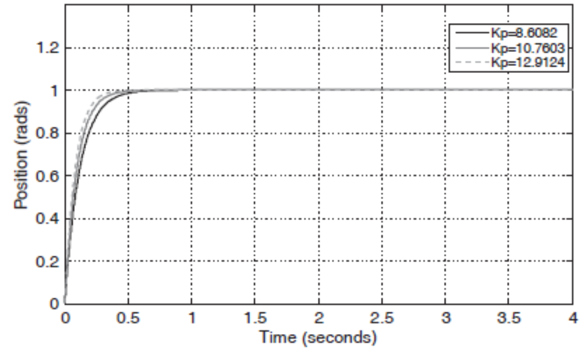
$Kp = 10.76, Kd = 0.006, \lambda = 0.508$



7.11 Simulation. Step responses with IOPID controller (T = 0.04s)



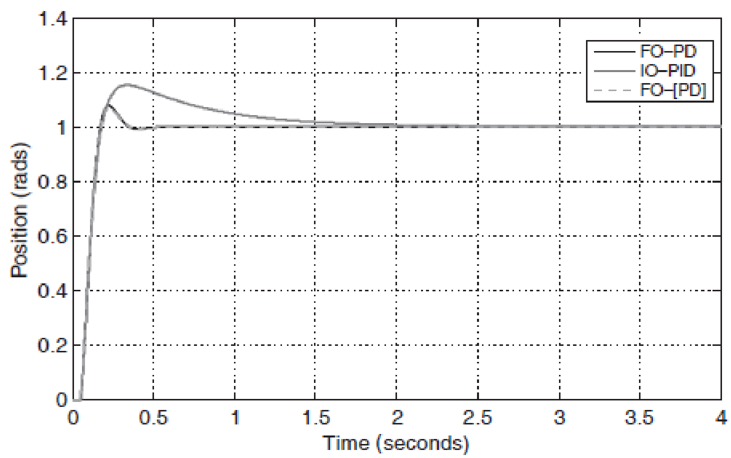
7.12 Simulation. Step responses with FOPD controller (T = 0.04s)



7.13 Simulation. Step responses with FO(PD) controller (T = 0.04s)

- Plant with time delay

$$P(s) = \frac{1}{s(0.4s+1)} e^{-\tau s}$$



Simulation. Step responses comparison with time delay of 0.05s (T = 0.04s)

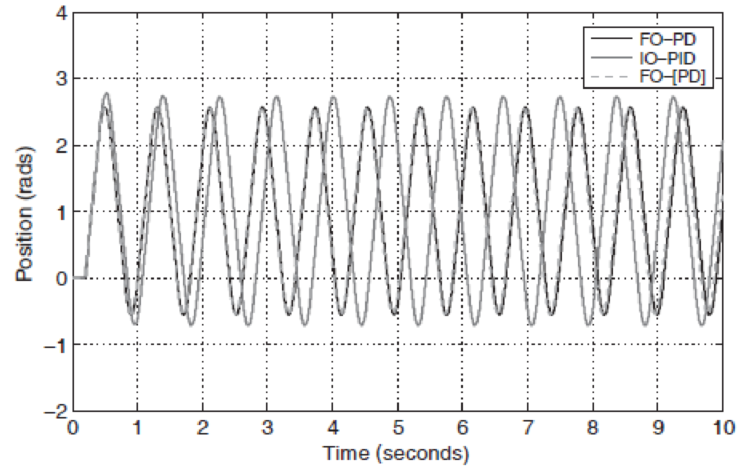
- Design specifications

$\omega_c = 10\, rad/s$

$\Phi_m = 70°$

Flat phase at ω_c

- Plant with backlash nonlinearity



Simulation. Step responses comparison with time delay of 0.2s (T = 0.04s)

Basic idea of time-Constant robustness

Use the gradient of the phase margin to time constant
and the gradient of the gain crossover frequency to time constant

The plant to be controlled

Second order position servo with time delay

$$P(s) = \frac{1}{s(Ts+1)} e^{-Ls}$$

The fractional order [PD] controller $(PD)^{\alpha}$

$$C(s) = (K_p + K_d s)^{\alpha}$$

The open loop transfer function

$$G(s) = C(s)P(s)$$

Phase margin

$$\angle C(j\omega) = \alpha \arctan(\frac{K_d \omega_c}{K_p}) - \arctan(T\omega_c) - \frac{\pi}{2} - L\omega_c = \phi - \pi$$

Gain crossover frequency specification

$$|G(j\omega)| = \frac{[(K_d\omega_c)^2 + {K_p}^2]^{\alpha/2}}{\omega_c\sqrt{1 + T^2{\omega_c}^2}} = 1$$

Robustness to time-constant variations

How ?

Quantitatively evaluation of the robustness

$$-\left.\frac{\frac{\partial|G(j\omega)|}{\partial\omega}}{\frac{\partial|G(j\omega)|}{\partial T}}\right|_{(\omega_c,T_0)} = -\left.\frac{\frac{\partial\angle[G(j\omega)]}{\partial\omega}}{\frac{\partial\angle[G(j\omega)]}{\partial T}}\right|_{(\omega_c,T_0)}.$$

Why ?

Gain robustness to ω and T

$$\left.\frac{\partial|G(j\omega)|}{\partial\omega}\right|_{(\omega_c,T_0)}\Delta\omega+\left.\frac{\partial|G(j\omega)|}{\partial T}\right|_{(\omega_c,T_0)}\Delta T=0,$$

$$\frac{\Delta\omega}{\Delta T}=-\left.\frac{\frac{\partial|G(j\omega)|}{\partial\omega}}{\frac{\partial|G(j\omega)|}{\partial T}}\right|_{(\omega_c,T_0)},$$

Phase margin robustness to ω and T

$$\left.\frac{\partial\angle[G(j\omega)]}{\partial\omega}\right|_{(\omega_c,T_0)}\Delta\omega+\left.\frac{\partial\angle[G(j\omega)]}{\partial T}\right|_{(\omega_c,T_0)}\Delta T=0,$$

$$\frac{\Delta\omega}{\Delta T}=-\left.\frac{\frac{\partial\angle[G(j\omega)]}{\partial\omega}}{\frac{\partial\angle[G(j\omega)]}{\partial T}}\right|_{(\omega_c,T_0)}.$$

- The derivation for computation of the solution existence range

$$\left.\frac{\partial|G(j\omega)|}{\partial\omega}\right|_{(\omega_c,T_0)}$$
$$=\frac{(K_d^2\omega_c^2+K_p^2)^{\frac{\alpha}{2}}\alpha K_d^2}{(K_d^2\omega_c^2+K_p^2)(1+T_0^2\omega_c^2)^{\frac{1}{2}}}-\frac{(K_d^2\omega_c^2+K_p^2)^{\frac{\alpha}{2}}}{\omega_c^2(1+T_0^2\omega_c^2)^{\frac{1}{2}}}-\frac{(K_d^2\omega_c^2+K_p^2)^{\frac{\alpha}{2}}T_0^2}{(1+T_0^2\omega_c^2)^{\frac{3}{2}}},$$

$$\left.\frac{\partial|G(j\omega)|}{\partial T}\right|_{(\omega_c,T_0)}=-\frac{(K_d^2\omega_c^2+K_p^2)^{\frac{\alpha}{2}}T_0\omega_c}{(1+T_0^2\omega_c^2)^{\frac{3}{2}}}.$$

$$\left.\frac{\partial A}{\partial\omega}\right|_{(\omega_c,T_0)}=\frac{\alpha K_d}{K_p\left(1+\frac{K_d^2\omega_c^2}{K_p^2}\right)}-\frac{T_0}{1+T_0^2\omega_c^2}-L,$$

$$\left.\frac{\partial A}{\partial T}\right|_{(\omega_c,T_0)}=-\frac{\omega_c}{1+T_0^2\omega_c^2}.$$

Simplified two equations

$$\alpha \arctan(A) - \arctan(T_0\omega_c) + \pi/2 - L\omega_c - \phi = 0, \tag{8.20}$$

$$\left(T_0 L\omega_c^2 - 1 + \alpha\right) A^2 - \alpha T_0\omega_c A + T_0 L\omega_c^2 - 1 = 0. \tag{8.21}$$

Solution for 8.20

$$A_{01} = \frac{\alpha\omega_c T_0 + \sqrt{\Delta_1}}{2(L\omega_c T_0\omega_c - 1 + \alpha)},$$

$$A_{02} = \frac{\alpha\omega_c T_0 - \sqrt{\Delta_1}}{2(L\omega_c T_0\omega_c - 1 + \alpha)},$$

$$\Delta_1 = (\alpha\omega_c T_0)^2 - 4(L\omega_c T_0\omega_c - 1 + \alpha)(L\omega_c T_0\omega_c - 1).$$

Solution for 8.21

$$A_1 = \tan\left(\frac{\phi'}{\alpha}\right),$$

Three cases

$$L T_0 {\omega_c}^{2} > 1$$

$$L T_0 {\omega_c}^{2} = 1$$

$$L T_0 {\omega_c}^{2} < 1$$

3 plants with different time constants

$$P_1(s) = \frac{1}{s(0.0241s+1)}e^{-0.01s},$$

$$P_2(s) = \frac{1}{s(0.0265s+1)}e^{-0.01s},$$

$$P_3(s) = \frac{1}{s(0.0301s+1)}e^{-0.01s}.$$

The resulting controller and implementation

$$C(s) = (160.5248 + 4.2539s)^{0.8332}.$$

$$C(z) = \frac{160.5z^5 - 486z^4 + 547.6z^3 - 278z^2 + 59.83z - 3.732}{0.05676z^5 - 0.1163z^4 + 0.07297z^3 - 0.01014z^2 - 0.00302z + 0.0004798}.$$

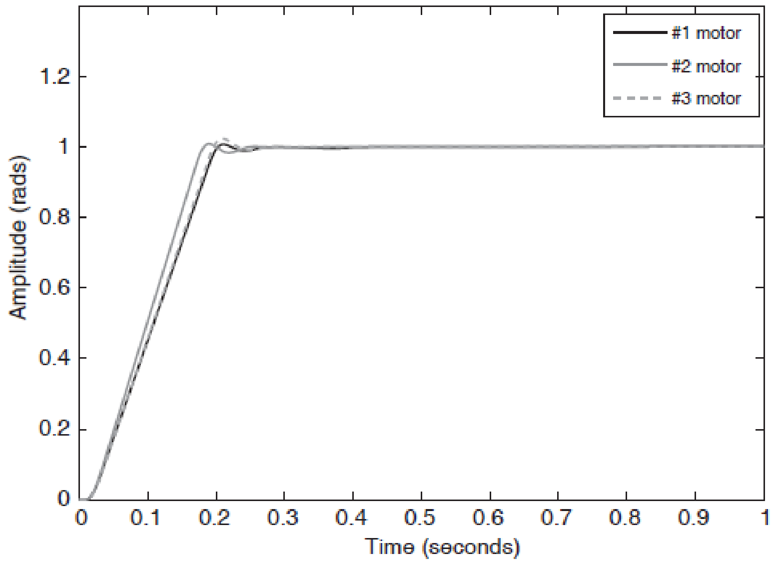


Figure: Step response comparison of the three motors models with different time constant

Conclusion

The overshoots on the three motors are all less than 2%.
Therefore, this tuning method guarantees system dynamic performance.

Scope of the book

The plant to be controlled

Fractional order model of *membrane charging*

Also: Fractional order position system

$$P(s) = \frac{1}{s(Ts^{\alpha}+1)} e^{-Ls}$$

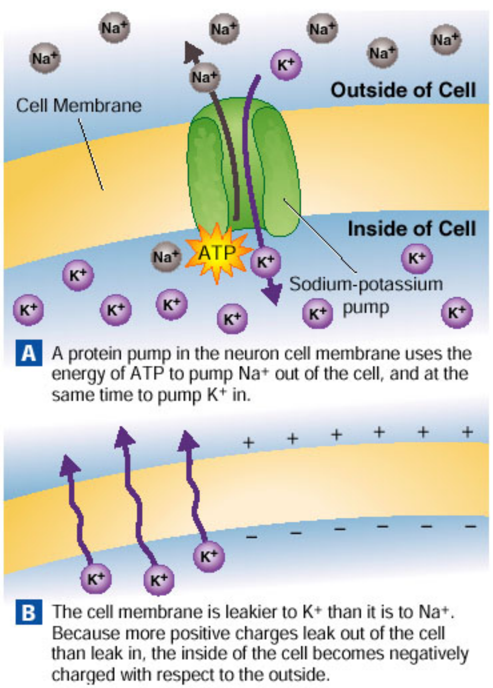


Figure: Cell membrane charging

Source: http://www.millerandlevine.com/chapter/35/898-899-rewrite.html

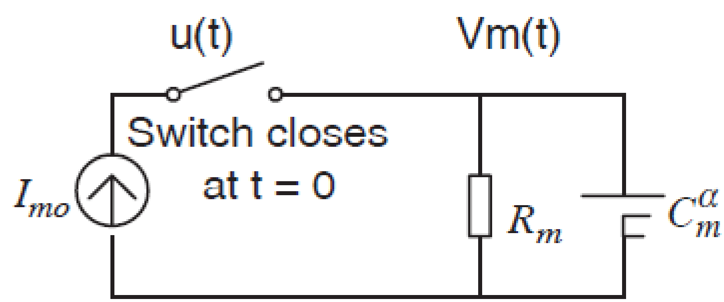


Figure: Membrane-charging circuit

$$C_m^{\alpha} \frac{d^{\alpha} V_m(t)}{dt^{\alpha}} + \frac{V_m(t)}{R_m} = I_{mo} u(t).$$

Time domain solution using Mittag-Leffler function

$$V_m(t) = I_{mo} R_m \frac{t^\alpha}{T} E_{\alpha,\alpha+1}\left(-\frac{t^\alpha}{T}\right).$$

G. M. Mittag-Leffler

Still using the FO PD controller

Refer to slide 7

$$C(s) = K_p(1 + K_d s^{\mu})$$

Still using the "flat phase" design specification

$$\angle[G(j\omega_c)] = \tan^{-1}\frac{\sin\frac{(1-\mu)\pi}{2} + K_d\omega_c^{\mu}}{\cos\frac{(1-\mu)\pi}{2}} + \frac{\mu\pi}{2} - \frac{\pi}{2} + \tan^{-1}\frac{1 + T\omega_c^{\alpha}\cos\frac{\alpha\pi}{2}}{T\omega_c^{\alpha}\sin\frac{\alpha\pi}{2}} = -\pi + \phi_m,$$

$$|G(j\omega_c)| = \frac{K_p\sqrt{\left(1 + K_d\omega_c^{\mu}\cos\frac{\mu\pi}{2}\right)^2 + \left(K_d\omega_c^{\mu}\sin\frac{\mu\pi}{2}\right)^2}}{\sqrt{\left(T\omega^{1+\alpha}\sin\frac{\alpha\pi}{2}\right)^2 + \left(\omega + T\omega^{1+\alpha}\cos\frac{\alpha\pi}{2}\right)^2}} = 1.$$

$$\left.\frac{\mathrm{d}(\angle(G(j\omega)))}{\mathrm{d}\omega}\right|_{\omega=\omega_c} = \frac{K_d}{1 + (K_d\omega_c)^2} - \frac{\alpha T\sin\frac{\alpha\pi}{2}\omega_c^{\alpha-1}}{1 + T^2\omega_c^{2\alpha} + 2T\omega_c^{\alpha}\cos\frac{\alpha\pi}{2}} = 0,$$

The design procedure

(1) Given ω_c, the gain crossover frequency.
(2) Given ϕ_m, the desired phase margin.
(3) Plot the curve 1, K_d with respect to μ, according to (9.11).
(4) Plot the curve 2, K_d with respect to μ, according to (9.14).
(5) Obtain the μ and K_d from the intersection point on the above two curves.
(6) Calculate K_p from (9.12).

Scope of the book

The plant to be controlled

FO position system

$$P(s) = \frac{1}{s(Ts^{\alpha} + 1)}$$

Gain and phase of the plant

$$\angle[P(j\omega)] = -\arctan\frac{T\omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)}{1 + T\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)} - \frac{\pi}{2}$$

$$|P(j\omega)| = \frac{1}{\sqrt{[T\omega^{1+\alpha}\sin\left(\frac{\alpha\pi}{2}\right)]^2 + [\omega + T\omega^{1+\alpha}\cos\left(\frac{\alpha\pi}{2}\right)]^2}}$$

Still using the FO [PD] controller $(PD)^{\alpha}$

$$C(s) = (K_p + K_d s)^{\alpha}$$

Still using the "flat phase" design specification

$$\begin{aligned}\angle[G_3(j\omega_c)] &= \mu \tan^{-1}(\omega_c K_{d3}) - \tan^{-1}\left(\frac{T\omega_c^{\alpha}\sin\frac{\alpha\pi}{2}}{1+T\omega_c^{\alpha}\cos\frac{\alpha\pi}{2}}\right) - \frac{\pi}{2}\\ &= -\pi + \phi_m,\end{aligned}$$

$$\begin{aligned}|G_3(j\omega_c)| &= |C_3(j\omega_c)||P(j\omega_c)|\\ &= \frac{K_{p3}(1+(K_{d3}\omega_c)^2)^{\frac{\mu}{2}}}{N}\\ &= 1,\end{aligned}$$

$$\begin{aligned}\left.\frac{\mathrm{d}(\angle(G_3(j\omega)))}{\mathrm{d}\omega}\right|_{\omega=\omega_c} &= \frac{\mu K_{d3}}{1+(K_{d3}\omega_c)^2} - \frac{\alpha T\omega_c^{\alpha-1}\sin\frac{\alpha\pi}{2}}{(T\omega_c^{\alpha}\sin\frac{\alpha\pi}{2})^2+(1+T\omega_c^{\alpha}\cos\frac{\alpha\pi}{2})^2}\\ &= 0,\end{aligned}$$

The design procedure

(1) Given parameters of the fractional order system to be controlled α and T.
(2) Given ω_c, the gain crossover frequency.
(3) Given ϕ_m, the desired phase margin.
(4) Plot the curve 1, K_{d3} with respect to μ, according to (10.10).
(5) Plot the curve 2, K_{d3} with respect to μ, according to (10.14).
(6) Obtain K_{d3} and μ from the intersection point on the above two curves.
(7) Calculate the K_{p3} from (10.11).

- Plant model

$$P(s) = \frac{1}{s(0.4s^{1.4} + 1)}$$

- Design specifications

$\omega_c = 10\ rad/s$
$\Phi_m = 70°$
Flat phase at ω_c

- IOPID parameters

$Kp = 18.29, Kd = -0.0846, Ki = 42.45$
Unstable

- FO PD

$Kp = 10.916, Kd = 0.61, \lambda = 1.189$

- FO [PD]

$Kp = 6.31, Kd = 0.9435, \lambda = 1.2$

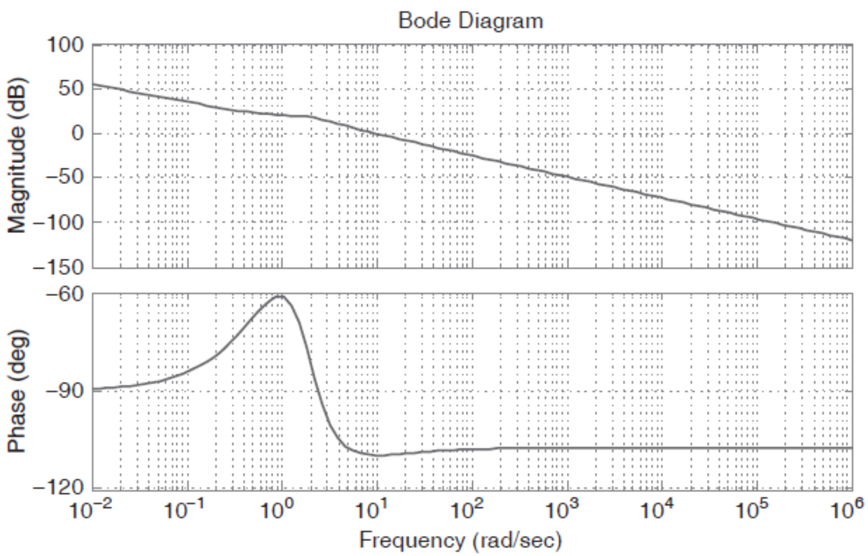


Figure: Bode plot with FO[PD] controller

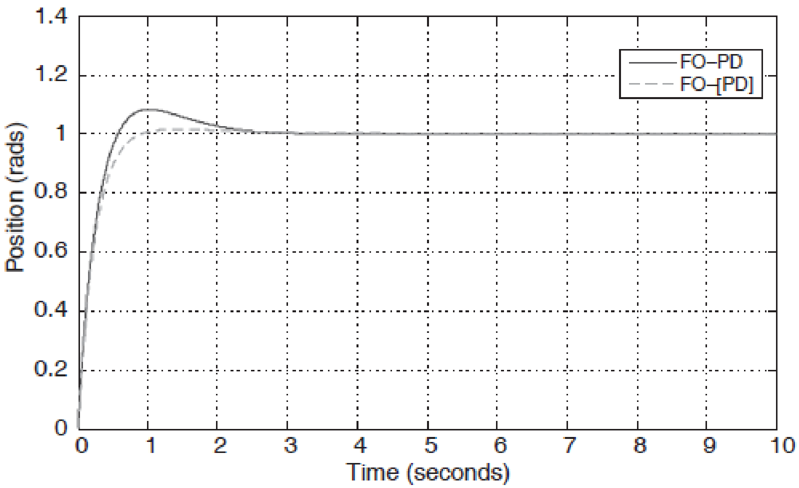


Figure: Step responses comparison with two FO controllers

Summary and review

- FO PD
- FO [PD]

For IO position system

- FO [PD]

Robust to time constant for IO position system with time delay

- FO PD
- FO [PD]

For FO position system

This is the end of session III

Questions?