



Turkish National Meeting on Automatic Control
(TOK 2013) , Sept. 25, 2013, Malatya, Turkey



A Tutorial on Fractional Order Motion Control

Part V: Fractional Order Disturbance Compensations

Prof. YangQuan Chen

Zhuo Li, PhD student

EECS, University of California, Merced

[ychen53](mailto:ychen53@ucmerced.edu), [zli32](mailto:zli32@ucmerced.edu)@ucmerced.edu

MESA Lab <http://mechatronics.ucmerced.edu>

- Fractional Order Motion Controls

John Wiley & Sons, Inc.

Hardcover, 454 pages, December 2012

- Dr. Ying Luo

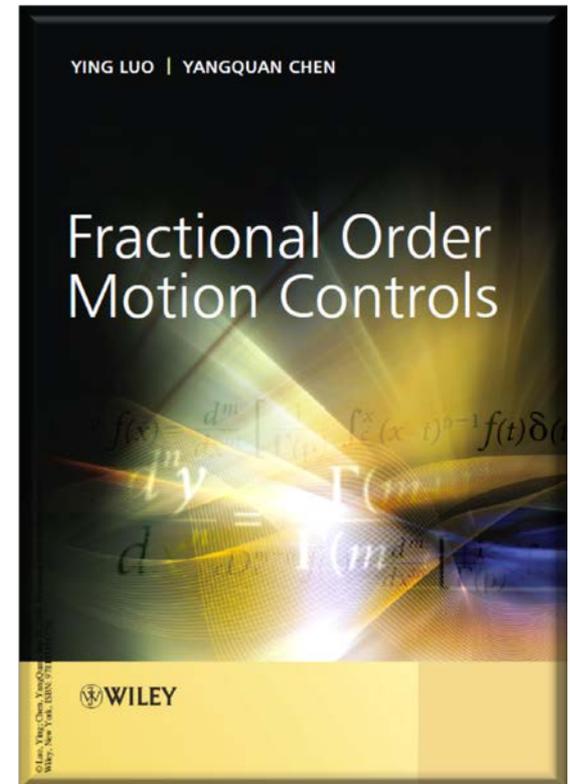
Automation Science and Engineering, South China
University of Technology, Guangzhou, PR China

- Dr. YangQuan Chen

Director, Mechatronics, Embedded Systems and
Automation (MESA) LAB

MEAM/EECS, School of Engineering,
University of California, Merced

Email: yqchen@ieee.org



- PART I FUNDAMENTALS OF FRACTIONAL CONTROLS
- 1 Introduction 3

- PART II FRACTIONAL ORDER VELOCITY SERVO
- 2 Fractional Order PI Controller Designs for Velocity Servo Systems 25
- 3 Tuning Fractional Order PI Controllers for Fractional Order Velocity Systems with Experimental Validation 41
- 4 Relay Feedback Tuning of Robust PID Controllers 59
- 5 Auto-Tuning of Fractional Order Controllers with ISO-Damping 73

- PART III FRACTIONAL ORDER POSITION SERVO
- 6 Fractional Order PD Controller Tuning for Position Systems 91
- 7 Fractional Order [PD] Controller Synthesis for Position Servo Systems 105
- 8 Time-Constant Robust Analysis and Design of Fractional Order [PD] Controller 123
- 9 Experimental Study of Fractional Order PD Controller Synthesis for Fractional Order Position Servo Systems 139
- 10 Fractional Order [PD] Controller Design and Comparison for Fractional Order Position Servo Systems 155

- PART IV STABILITY AND FEASIBILITY FOR FOPID DESIGN
- 11 Stability and Design Feasibility of Robust PID Controllers for FOPTD Systems 165
- 12 Stability and Design Feasibility of Robust FOPI Controllers for FOPTD Systems 187

- **PART V FRACTIONAL ORDER DISTURBANCE COMPENSATORS**
- **13 Fractional Order Disturbance Observer 211**
- **14 Fractional Order Adaptive Feed-forward Cancellation 223**
- **15 Fractional Order Robust Control for Cogging Effect 243**
- **16 Fractional Order Periodic Adaptive Learning Compensation 275**

- PART VI EFFECTS OF FRACTIONAL ORDER CONTROLS ON NONLINEARITIES
- 17 Fractional Order PID Control of A DC-Motor with Elastic Shaft 293
- 18 Fractional Order Ultra Low-Speed Position Servo 313
- 19 Optimized Fractional Order Conditional Integrator 329

- PART VII FRACTIONAL ORDER CONTROL APPLICATIONS
- 20 Lateral Directional Fractional Order Control of A Small Fixed-Wing UAV 345
- 21 Fractional Order PD Controller Synthesis and Implementation for HDD Servo System 369

- PART I FUNDAMENTALS OF FRACTIONAL CONTROLS
- 1 Introduction 3

- PART II FRACTIONAL ORDER VELOCITY SERVO
- 2 Fractional Order PI Controller Designs for Velocity Servo Systems 25
- 3 Tuning Fractional Order PI Controllers for Fractional Order Velocity Systems with Experimental Validation 41
- 4 Relay Feedback Tuning of Robust PID Controllers 59
- 5 Auto-Tuning of Fractional Order Controllers with ISO-Damping 73

- PART III FRACTIONAL ORDER POSITION SERVO
- 6 Fractional Order PD Controller Tuning for Position Systems 91
- 7 Fractional Order [PD] Controller Synthesis for Position Servo Systems 105
- 8 Time-Constant Robust Analysis and Design of Fractional Order [PD] Controller 123
- 9 Experimental Study of Fractional Order PD Controller Synthesis for Fractional Order Position Servo Systems 139
- 10 Fractional Order [PD] Controller Design and Comparison for Fractional Order Position Servo Systems 155

- PART IV STABILITY AND FEASIBILITY FOR FOPID DESIGN
- 11 Stability and Design Feasibility of Robust PID Controllers for FOPTD Systems 165
- 12 Stability and Design Feasibility of Robust FOPI Controllers for FOPTD Systems 187

- **PART V FRACTIONAL ORDER DISTURBANCE COMPENSATORS**
- **13 Fractional Order Disturbance Observer 211**
- **14 Fractional Order Adaptive Feed-forward Cancellation 223**
- **15 Fractional Order Robust Control for Cogging Effect 243**
- **16 Fractional Order Periodic Adaptive Learning Compensation 275**

- PART VI EFFECTS OF FRACTIONAL ORDER CONTROLS ON NONLINEARITIES
- 17 Fractional Order PID Control of A DC-Motor with Elastic Shaft 293
- 18 Fractional Order Ultra Low-Speed Position Servo 313
- 19 Optimized Fractional Order Conditional Integrator 329

- PART VII FRACTIONAL ORDER CONTROL APPLICATIONS
- 20 Lateral Directional Fractional Order Control of A Small Fixed-Wing UAV 345
- 21 Fractional Order PD Controller Synthesis and Implementation for HDD Servo System 369

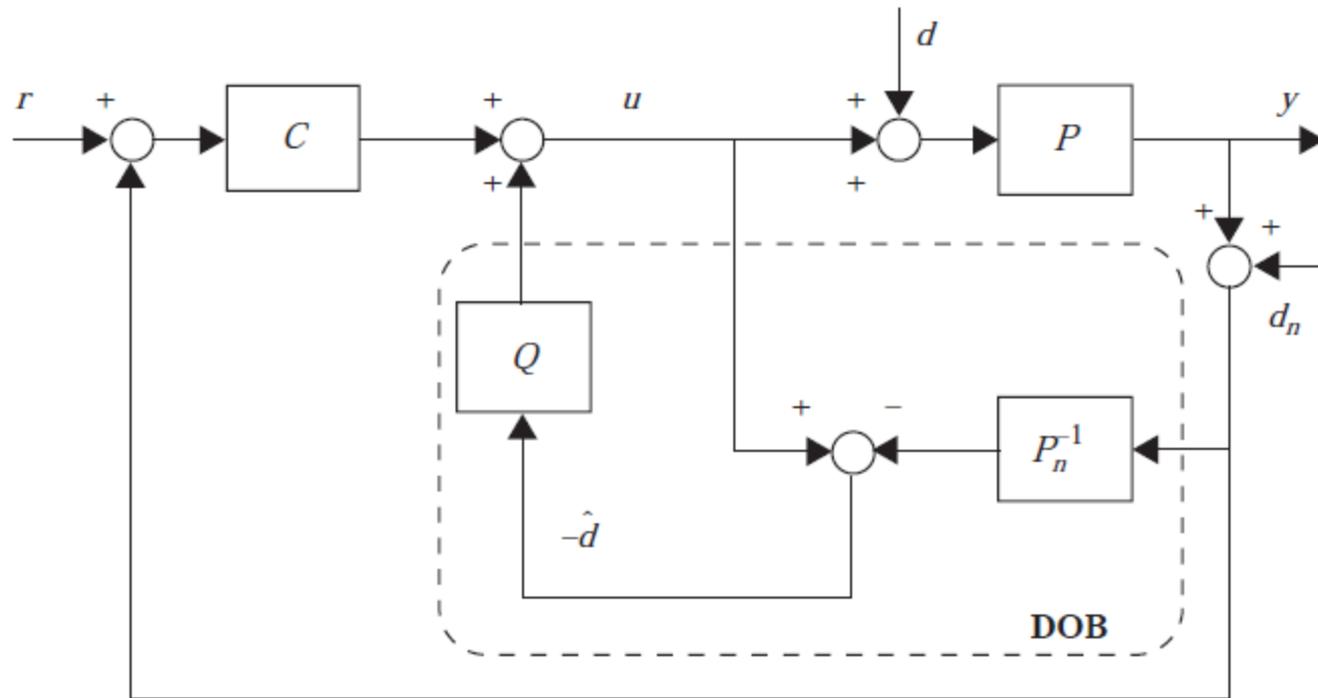


Figure: The conventional disturbance observer block-diagram.

Design parameters

n_d : the number of pure time delays of the control signal

n_Q : the relative degree of Q-filter

ω_Q : the cutoff frequency of Q-filter

The error transfer function

Without DOB

$$S(j\omega) = \frac{1}{1 + PC}$$

With DOB

$$S(j\omega) = \frac{1}{1 + PC + \delta_{PC}}$$
$$\delta_{PC} = \frac{PP_n^{-1}Q + z^{-n_d}QPC}{1 - z^{-n_d}Q}$$

Effects of the design parameters

$n_Q \uparrow \Rightarrow$ phase margin \downarrow

$\omega_Q \uparrow \Rightarrow$ phase margin \downarrow

A compromise must be made between the disturbance attenuation performance and the robustness of the original system.

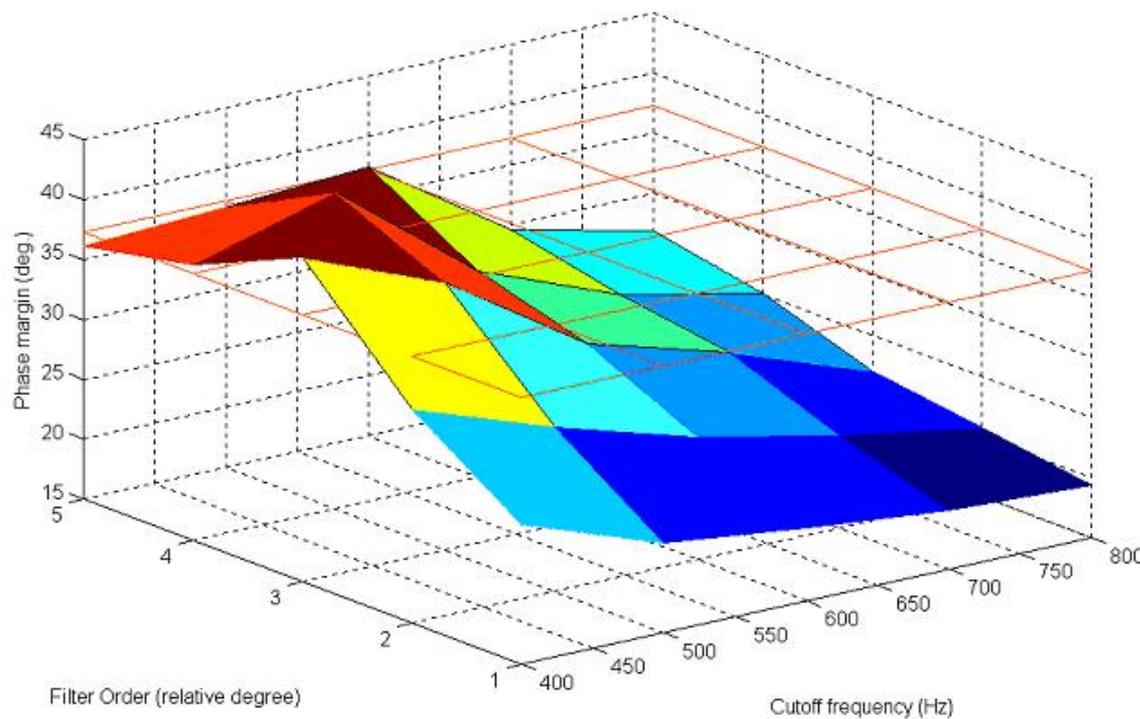


Figure: An illustration of the phase margin (PM) as a function of n_Q and ω_Q in DOB

A trade-off solution:

Rule-based switched low pass filtering with varying relative degree
 Tuning parameter: n_Q

(19) United States	
(12) Patent Application Publication	(10) Pub. No.: US 2001/0036026 A1
Chen et al.	(43) Pub. Date: Nov. 1, 2001
<hr/>	
(54) EFFICIENT SENSORLESS ROTATIONAL VIBRATION AND SHOCK COMPENSATOR FOR HARD DISK DRIVES WITH HIGHER TPI	Related U.S. Application Data
	(63) Non-provisional of provisional application No. 60/184,718, filed on Feb. 24, 2000.
(75) Inventors: YangQuan Chen , Singapore (SG); MingZhong Ding , Singapore (SG); LeeLing Tan , Singapore (SG); KianKeong Ooi , Singapore (SG)	Publication Classification
	(51) Int. Cl.⁷ G11B 27/36; G11B 21/02
	(52) U.S. Cl. 360/31; 360/77.02; 360/75; 360/69

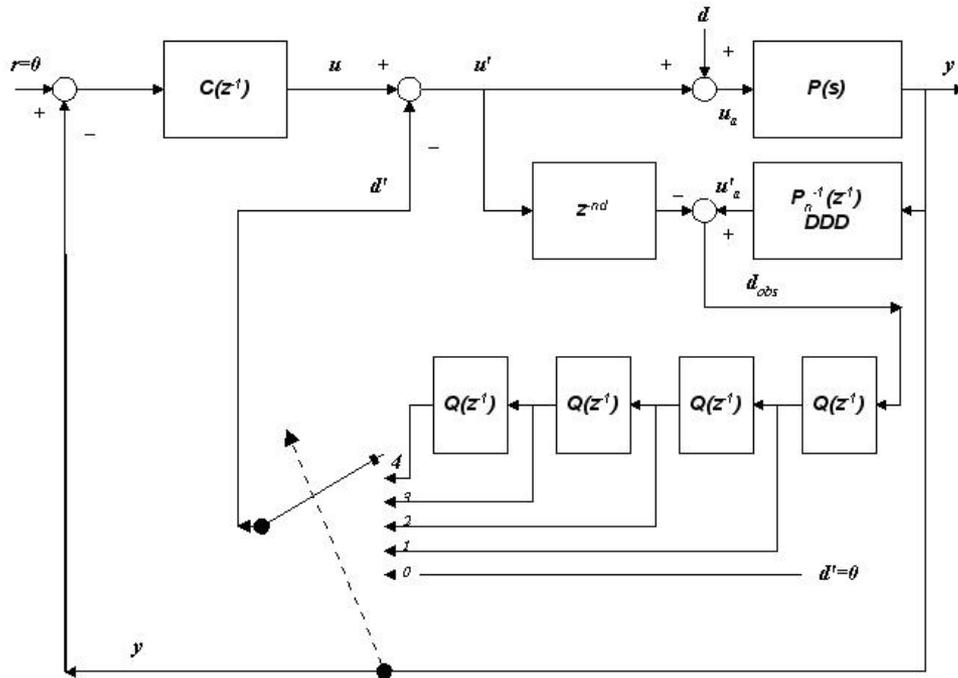
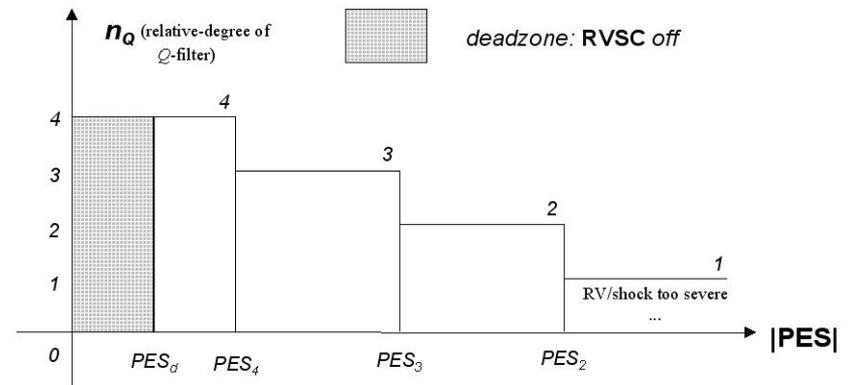


Figure: Q-filter in DOB with a varying relative degree

Figure: A switching policy for the relative degrees of the Q-filter in DOB.



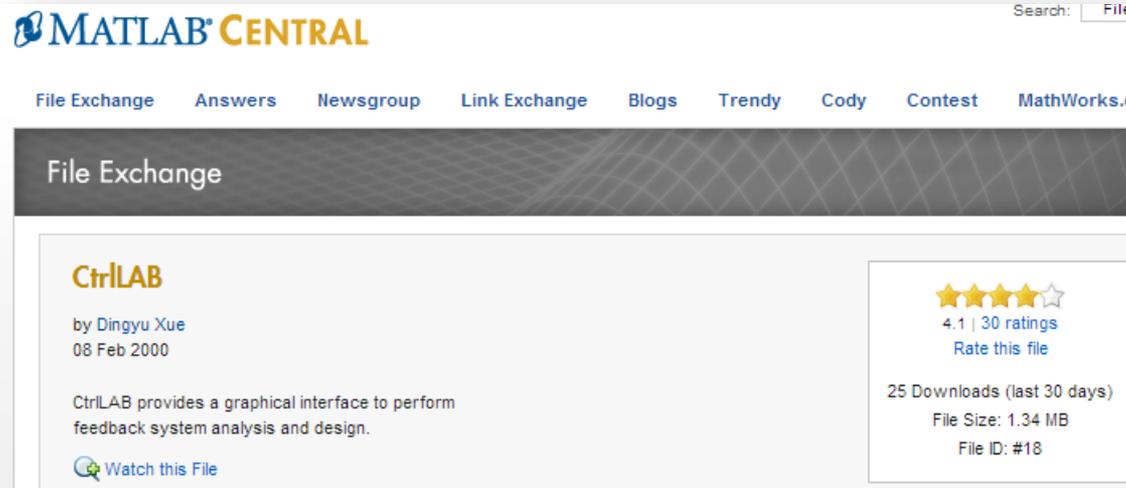
The proposed trade-off solution:

Fractional order DOB using a fractional order Q filter

$$Q(s) = \frac{1}{(\tau s + 1)^{n_Q}}, \{n_Q | n_Q \in Q, n_Q > 0\}$$

The implementation

Stable minimum-phase frequency domain fitting



The screenshot shows the MATLAB Central File Exchange interface. At the top, the MATLAB Central logo is visible, along with a search bar and navigation links for File Exchange, Answers, Newsgroup, Link Exchange, Blogs, Trendy, Cody, Contest, and MathWorks.com. The main content area is titled "File Exchange" and features a card for the file "CtrlLAB". The card includes the file name "CtrlLAB", the author "by Dingyu Xue", and the date "08 Feb 2000". A description states: "CtrlLAB provides a graphical interface to perform feedback system analysis and design." Below the description is a "Watch this File" button. To the right of the description, there is a star rating of 4.1 out of 5, based on 30 ratings, with a "Rate this file" link. Below the rating, it shows "25 Downloads (last 30 days)", "File Size: 1.34 MB", and "File ID: #18".

- PART I FUNDAMENTALS OF FRACTIONAL CONTROLS
- 1 Introduction 3

- PART II FRACTIONAL ORDER VELOCITY SERVO
- 2 Fractional Order PI Controller Designs for Velocity Servo Systems 25
- 3 Tuning Fractional Order PI Controllers for Fractional Order Velocity Systems with Experimental Validation 41
- 4 Relay Feedback Tuning of Robust PID Controllers 59
- 5 Auto-Tuning of Fractional Order Controllers with ISO-Damping 73

- PART III FRACTIONAL ORDER POSITION SERVO
- 6 Fractional Order PD Controller Tuning for Position Systems 91
- 7 Fractional Order [PD] Controller Synthesis for Position Servo Systems 105
- 8 Time-Constant Robust Analysis and Design of Fractional Order [PD] Controller 123
- 9 Experimental Study of Fractional Order PD Controller Synthesis for Fractional Order Position Servo Systems 139
- 10 Fractional Order [PD] Controller Design and Comparison for Fractional Order Position Servo Systems 155

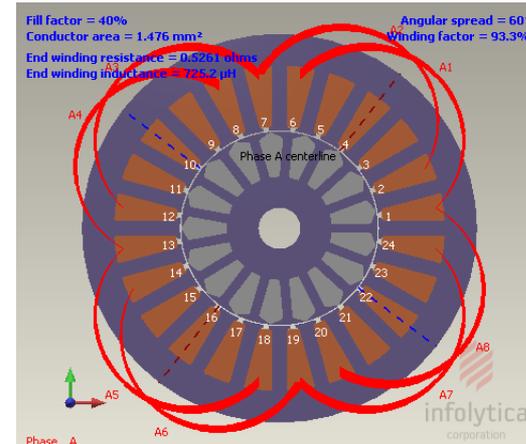
- PART IV STABILITY AND FEASIBILITY FOR FOPID DESIGN
- 11 Stability and Design Feasibility of Robust PID Controllers for FOPTD Systems 165
- 12 Stability and Design Feasibility of Robust FOPI Controllers for FOPTD Systems 187

- **PART V FRACTIONAL ORDER DISTURBANCE COMPENSATORS**
- **13 Fractional Order Disturbance Observer 211**
- **14 Fractional Order Adaptive Feed-forward Cancellation 223**
- **15 Fractional Order Robust Control for Cogging Effect 243**
- **16 Fractional Order Periodic Adaptive Learning Compensation 275**

- PART VI EFFECTS OF FRACTIONAL ORDER CONTROLS ON NONLINEARITIES
- 17 Fractional Order PID Control of A DC-Motor with Elastic Shaft 293
- 18 Fractional Order Ultra Low-Speed Position Servo 313
- 19 Optimized Fractional Order Conditional Integrator 329

- PART VII FRACTIONAL ORDER CONTROL APPLICATIONS
- 20 Lateral Directional Fractional Order Control of A Small Fixed-Wing UAV 345
- 21 Fractional Order PD Controller Synthesis and Implementation for HDD Servo System 369

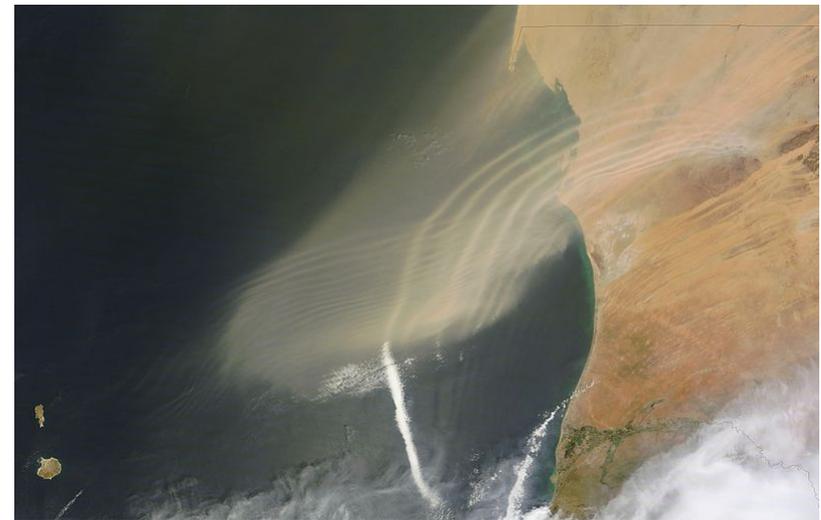
- Periodic disturbance
 - Telecommunication
 - Building vibration
- Periodic disturbance in motion control
 - Repeatable spindle motor runout
 - Cogging force



Cogging effect



Tacoma narrow bridge, WA, 1941



Atmosphere wave

http://en.wikipedia.org/wiki/Atmospheric_wave

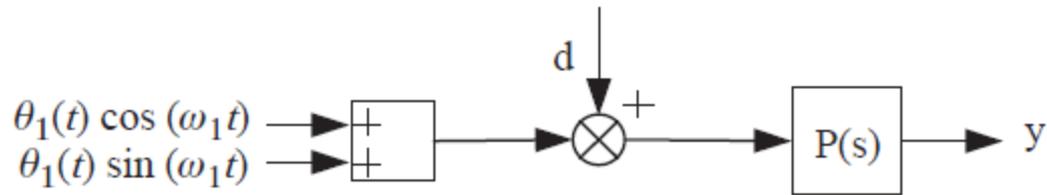


Figure: Fractional order adaptive feed-forward cancellation

The disturbance

$$d(t) = A \sin(\omega_1 t + \phi) = a_1 \cos(\omega_1 t) + b_1 \sin(\omega_1 t).$$

The control input

$$u(t) = \theta_1(t) \cos(\omega_1 t) + \theta_2(t) \sin(\omega_1 t),$$

The plant output

$$y(t) = \mathbb{L}^{-1}[\mathbb{L}((\theta_1(t) - \theta_1^*) \cos(\omega_1 t) + (\theta_2(t) - \theta_2^*) \sin(\omega_1 t))P(s)],$$

The nominal values

$$\theta_1^* = -a_1, \theta_2^* = -b_1,$$

The adaptive law

Using Caputo definition for fractional derivative

$${}_0D_t^\alpha \theta_1(t) = -gy(t) \cos(\omega_1 t),$$

$${}_0D_t^\alpha \theta_2(t) = -gy(t) \sin(\omega_1 t),$$

The derivation of the adaptive law

$${}_0D_t^\alpha \theta_1(t) = -\frac{g}{2} y(t) (e^{j\omega_1 t} + e^{-j\omega_1 t}),$$

$${}_0D_t^\alpha \theta_2(t) = -\frac{jg}{2} y(t) (e^{-j\omega_1 t} - e^{j\omega_1 t}).$$

The Laplace transform

Using the time shifting property

$$\Theta_1(s) = -\frac{g}{2s^\alpha} (Y(s + j\omega_1) + Y(s - j\omega_1)),$$

$$\Theta_2(s) = -\frac{jg}{2s^\alpha} (Y(s - j\omega_1) - Y(s + j\omega_1)).$$

The FO adaptive compensator

$$\begin{aligned}
 U(s) &= \frac{1}{2}(\Theta_1(s - j\omega_1) + \Theta_1(s + j\omega_1)) \\
 &\quad + \frac{j}{2}(\Theta_2(s - j\omega_1) - \Theta_2(s + j\omega_1)) \\
 &= -\frac{g}{2} \left(\frac{1}{(s + j\omega_1)^\alpha} + \frac{1}{(s - j\omega_1)^\alpha} \right) Y(s) \\
 &= -gC_{IMP}(s)Y(s),
 \end{aligned}$$

Same !

The FO internal model principle

$$C_{IMP}(s) = \frac{(s - j\omega_1)^\alpha + (s + j\omega_1)^\alpha}{2(s^2 + \omega^2)^\alpha}$$

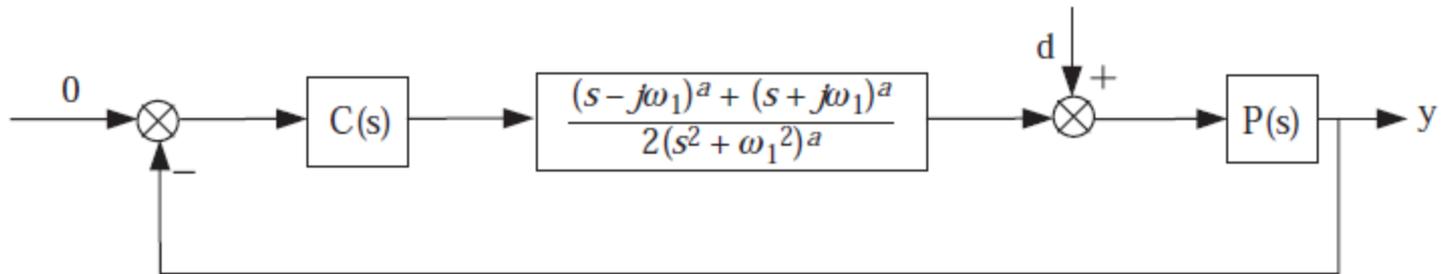


Figure: Fractional order internal model principle equivalence of the fractional order adaptive feed-forward cancellation, with $C(s)=g$

The plant and the disturbance

$$P(s) = \frac{s + 2}{(s + 1)(s + 3)},$$

$$d(t) = \sin(0.1t) - 0.2 \sin(0.3t),$$

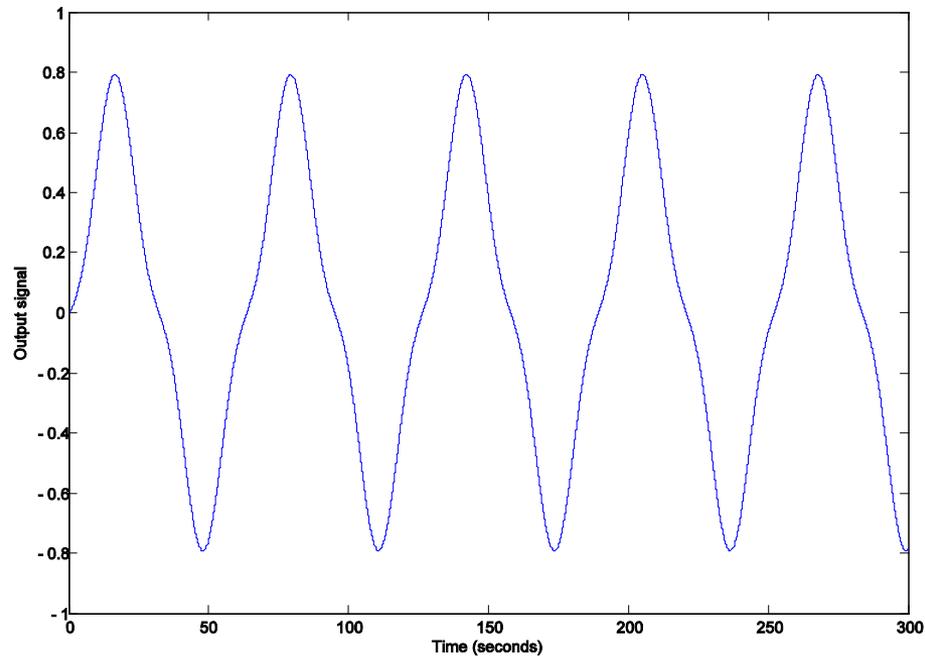


Figure: Plant output without compensation

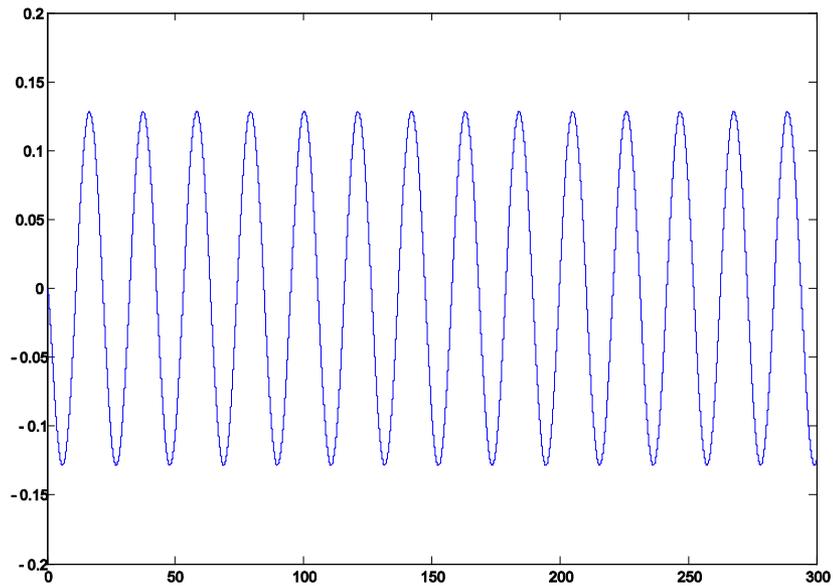


Figure: Plant output with fixed nominal compensation

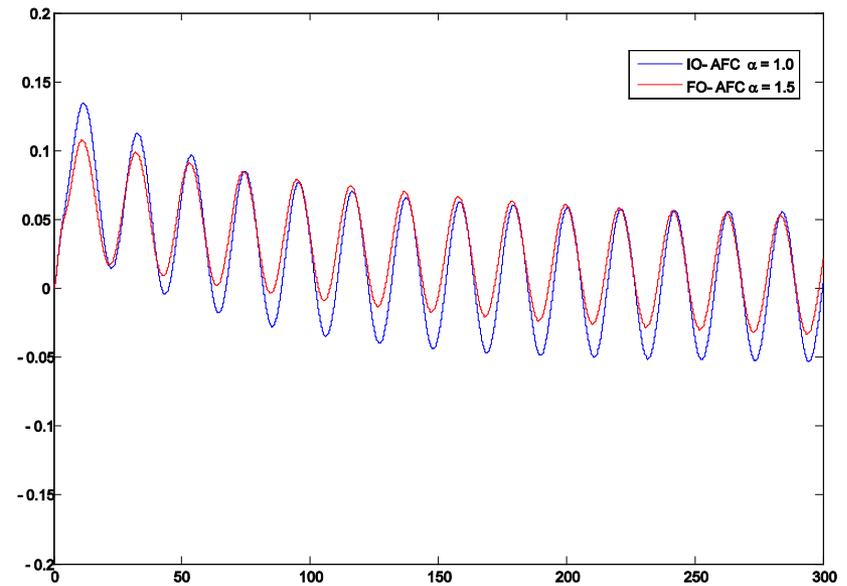
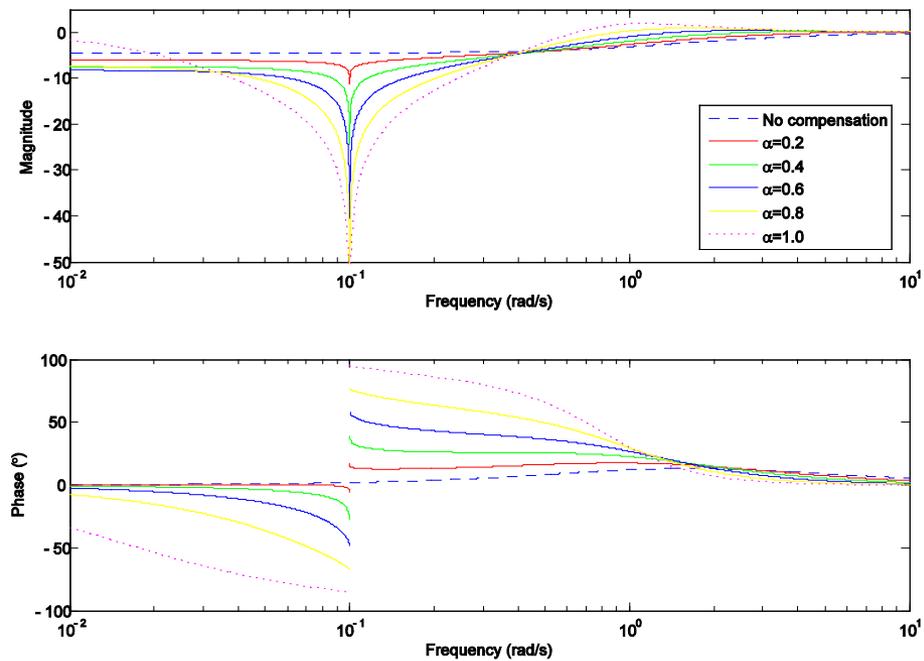


Figure: Plant output with IOAFC compensation and $\alpha = 1.5$ FOAFC

The sensitivity function

$$G_s(s) = \frac{1}{1 + C(s)C_{IMP}(s)P(s)}$$

Figure: Bode plots of the sensitivity function with $\omega_1 = 0.1$ and $\alpha \in (0, 1]$

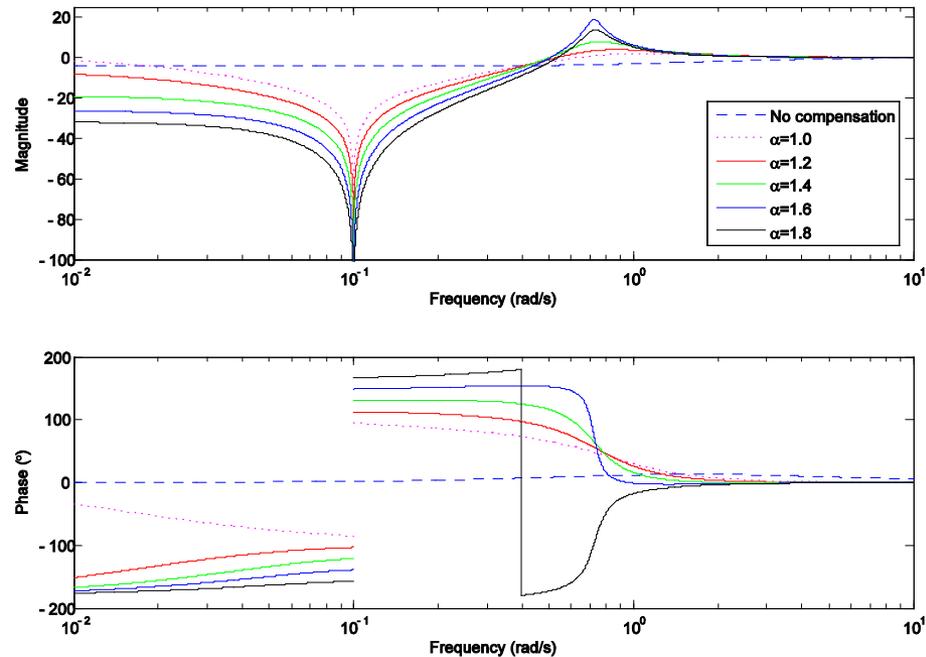


Figure: Bode plots of the sensitivity function with $\omega_1 = 0.1$ and $\alpha \in (1, 2]$

Comparison

$\alpha \in (0, 1]$, FOAFC is better than IOAFC

$\alpha \in (1, 2)$, IOAFC is better than FOAFC

- PART I FUNDAMENTALS OF FRACTIONAL CONTROLS
- 1 Introduction 3

- PART II FRACTIONAL ORDER VELOCITY SERVO
- 2 Fractional Order PI Controller Designs for Velocity Servo Systems 25
- 3 Tuning Fractional Order PI Controllers for Fractional Order Velocity Systems with Experimental Validation 41
- 4 Relay Feedback Tuning of Robust PID Controllers 59
- 5 Auto-Tuning of Fractional Order Controllers with ISO-Damping 73

- PART III FRACTIONAL ORDER POSITION SERVO
- 6 Fractional Order PD Controller Tuning for Position Systems 91
- 7 Fractional Order [PD] Controller Synthesis for Position Servo Systems 105
- 8 Time-Constant Robust Analysis and Design of Fractional Order [PD] Controller 123
- 9 Experimental Study of Fractional Order PD Controller Synthesis for Fractional Order Position Servo Systems 139
- 10 Fractional Order [PD] Controller Design and Comparison for Fractional Order Position Servo Systems 155

- PART IV STABILITY AND FEASIBILITY FOR FOPID DESIGN
- 11 Stability and Design Feasibility of Robust PID Controllers for FOPTD Systems 165
- 12 Stability and Design Feasibility of Robust FOPI Controllers for FOPTD Systems 187

- **PART V FRACTIONAL ORDER DISTURBANCE COMPENSATORS**
- **13 Fractional Order Disturbance Observer 211**
- **14 Fractional Order Adaptive Feed-forward Cancellation 223**
- **15 Fractional Order Robust Control for Cogging Effect 243**
- **16 Fractional Order Periodic Adaptive Learning Compensation 275**

- PART VI EFFECTS OF FRACTIONAL ORDER CONTROLS ON NONLINEARITIES
- 17 Fractional Order PID Control of A DC-Motor with Elastic Shaft 293
- 18 Fractional Order Ultra Low-Speed Position Servo 313
- 19 Optimized Fractional Order Conditional Integrator 329

- PART VII FRACTIONAL ORDER CONTROL APPLICATIONS
- 20 Lateral Directional Fractional Order Control of A Small Fixed-Wing UAV 345
- 21 Fractional Order PD Controller Synthesis and Implementation for HDD Servo System 369

The cogging effect

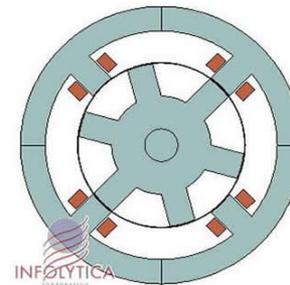
- Trouble maker of PMSM (permanent magnetic synchronous motor)
- Especially at low speed
- Generated by the magnetic attraction between the rotor and the stator
- Appear at $N_{slot-pp}f_s$

Modeling the cogging force

- Multi-harmonic Fourier expansion

$$F_{cogging} = \sum_{i=1}^{\infty} A_i \sin(\omega_i x + \varphi_i)$$

- State dependent



The plant model

$$\dot{\theta}(t) = v(t),$$

$$\dot{v}(t) = u - \frac{a(\theta)}{J} - T_l - B_1 v,$$

$$u = \frac{1}{J} T_m, T_l = \frac{1}{J} T_l, B_1 = \frac{B}{J},$$

θ : angular position, v : velocity; $a(\theta)$ unknown position-dependent cogging disturbance; J : moment of inertia; T_m : electromagnetic torque; T_l : load torque; B : viscous friction coefficient

The displacement

$$e_\theta(t) = \theta_d(t) - \theta(t)$$
$$e_v(t) = \dot{e}_\theta(t) = v_d(t) - v(t)$$

The adaptive compensator (AC)

$$u(t) = \dot{v}_d(t) + T_F + \frac{\hat{a}(t)}{J} + \alpha m(t) + \gamma e_v(t),$$

where

$$m(t) := \gamma e_\theta(t) + e_v(t),$$

The adaptive law for $\hat{a}(t)$

$$\hat{a}(t) = z - \mu v$$

$${}_0D_t^\nu z(t) = \mu[\dot{v}_d(t) + \alpha m(t) + \gamma e_v(t)] + \frac{e_v(t)}{J},$$

$$\begin{cases} v = 1, IOAC \\ v \in (0, 1), FOAC \end{cases}$$

IOAC stability theorem

If the integer order adaptive compensation is used, the equilibrium points e_θ and e_v are bounded as $t \rightarrow \infty$.

FOAC stability theorem

If the parameters α , γ , and μ , are chosen to ensure

$$|\arg(\omega_i)| > \lambda \frac{\pi}{2}$$

the equilibrium points e_θ and e_v are bounded as $t \rightarrow \infty$,

Where ω_i are the roots of

$$w^{2pq+p^2} + aw^{pq+p^2} + bw^{pq} + dw^{p^2} + c = 0,$$

The proof is available in the book

The applied cogging force

$$F_{cogging} = 2 \cos(6\theta) + \cos(12\theta) + 0.5\cos(18\theta)$$

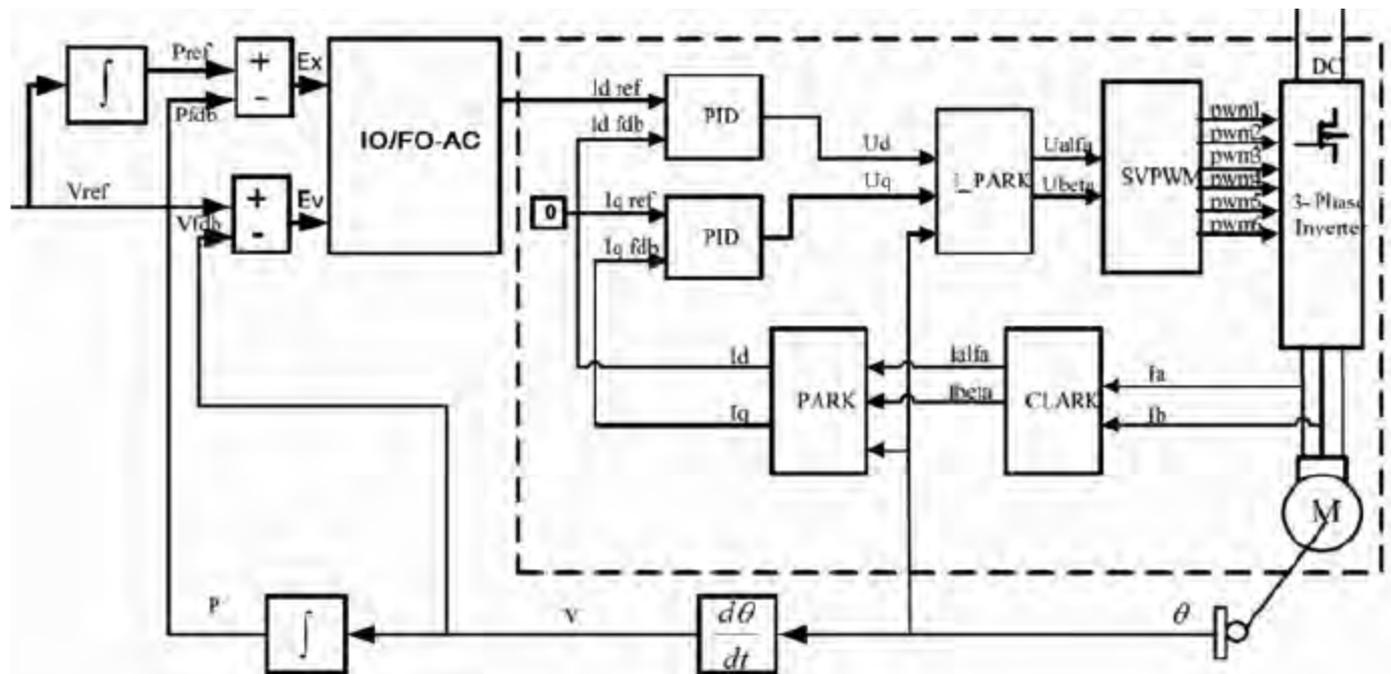


Figure: the block diagram of PMSM servo simulation system with the IO/FOAC for cogging effect compensation

Table: PMSM Specifications

Rated power	1.64 Kw	Rated speed	2000 rpm
Rated torque	8 Nm	Stator resistance	2.125 Ω
Stator inductance	11.6 mH	Magnetic flux	0.387 Wb
Number of poles	6	Moment of inertia	0.00289 kgm ²
Friction coefficient	0.0003 Nms		

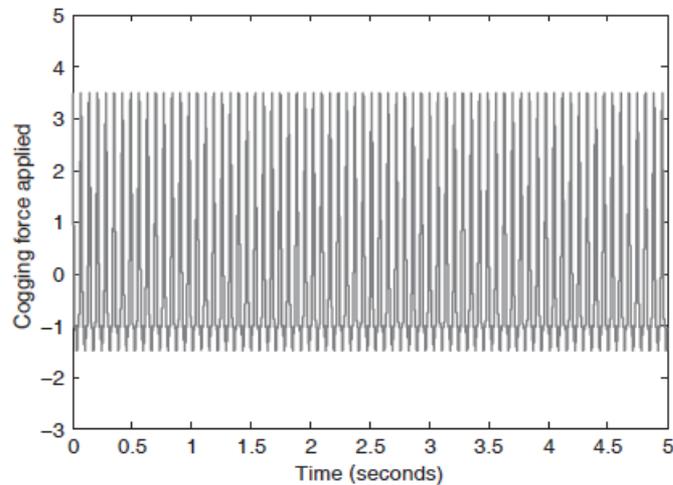
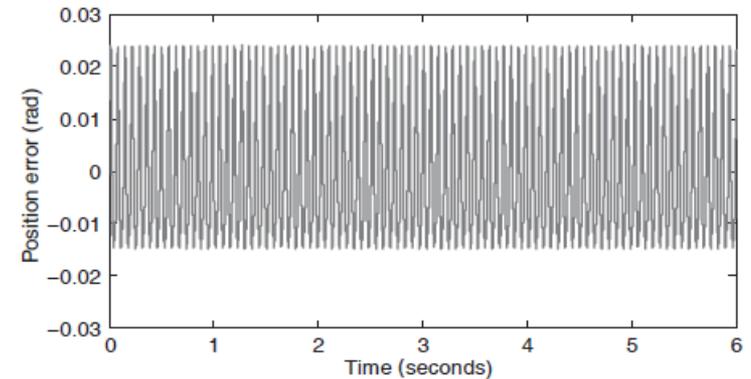
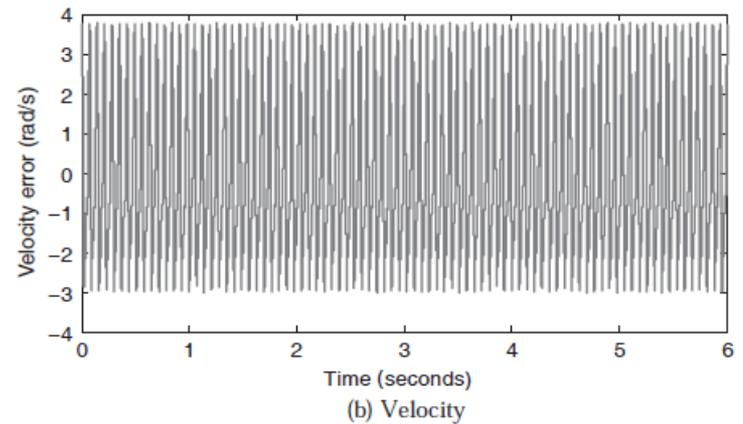


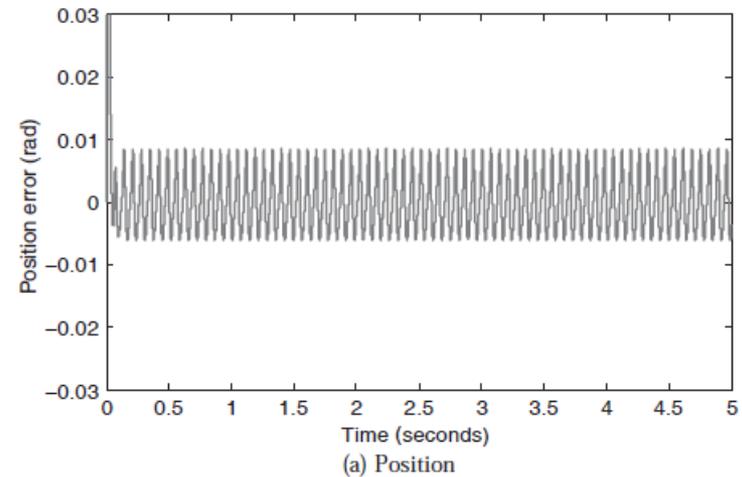
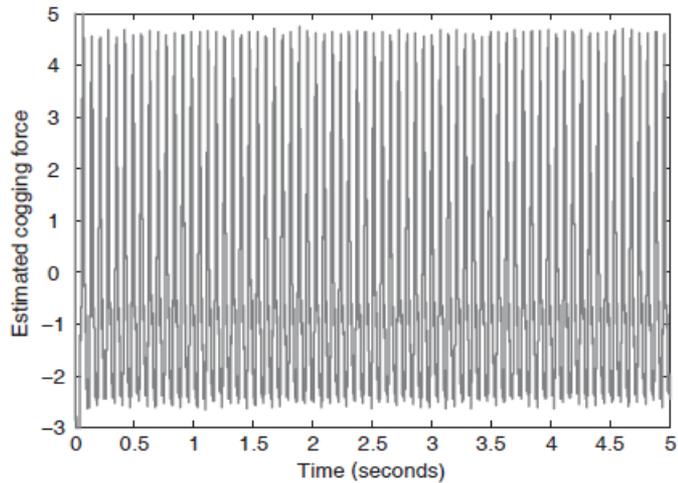
Figure 15.2 Simulation. Cogging force applied



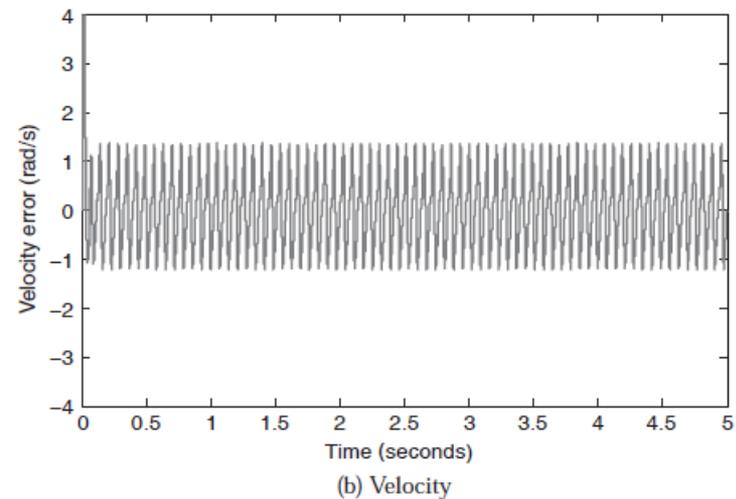
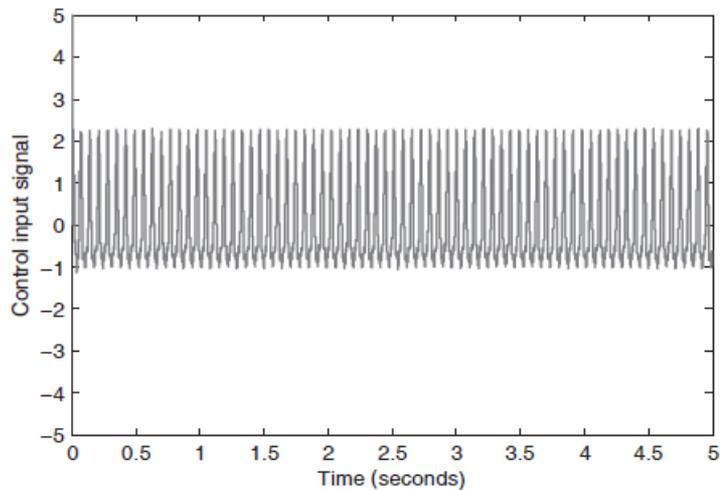
(a) Position



(b) Velocity

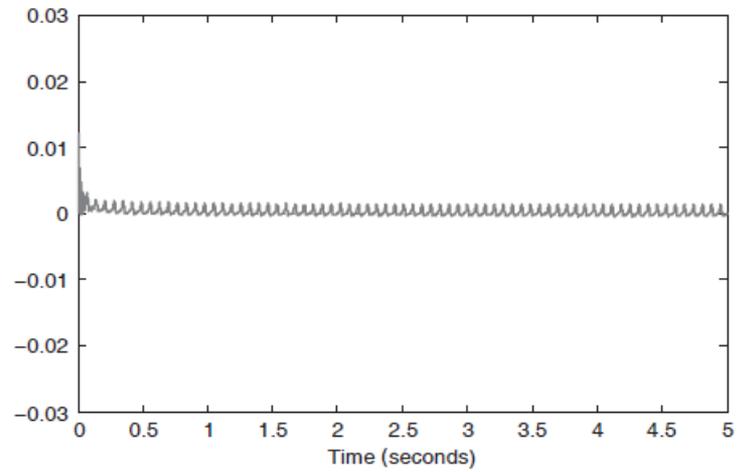
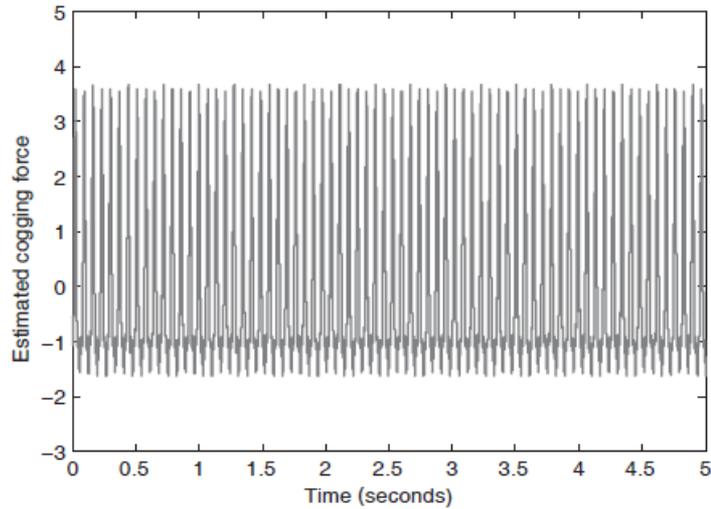


15.4 Simulation. Estimated cogging force with IOAC ($\nu = 1$)



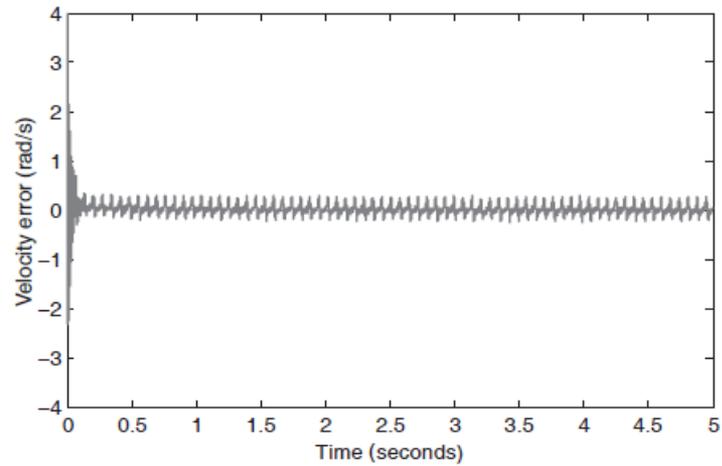
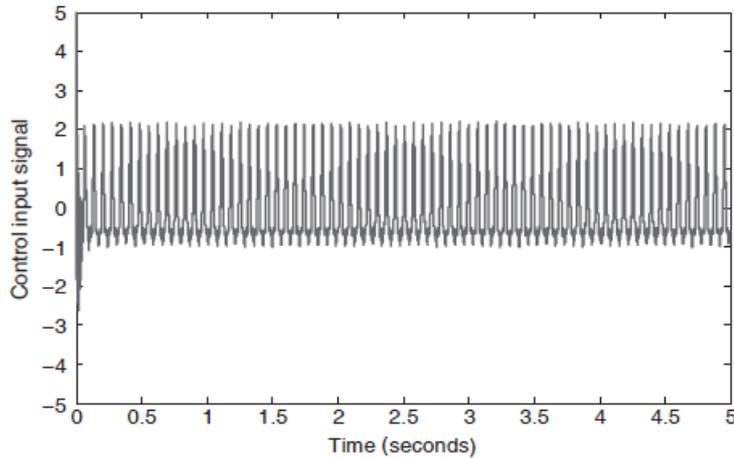
Simulation. Constant reference speed tracking control input signal

Simulation. Constant reference speed tracking errors with IOAC ($\nu = 1$)



(a) Position

15.7 Simulation. Estimated cogging force with FOAC ($\nu = 0.5$)



(b) Velocity

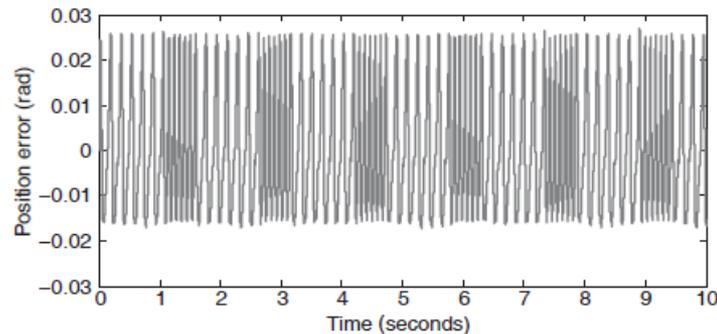
Simulation. Constant reference speed tracking control input signal

Simulation. Constant reference speed tracking errors with FOAC ($\nu = 0.5$)

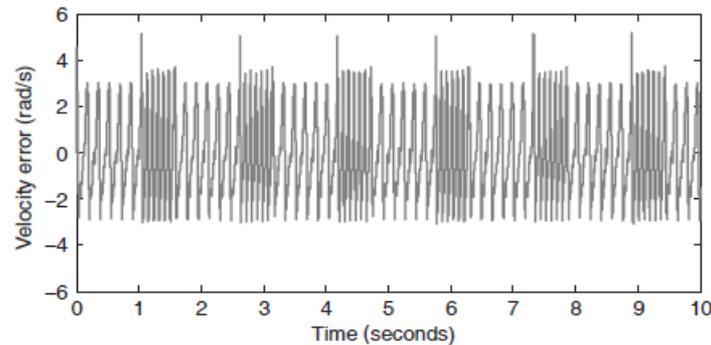
The varying reference speed

$$s_d(t) = \int_0^t v_d(\tau) d\tau,$$

$$v_d(t) = \begin{cases} 2 \text{ rad/s} & \text{if } js_p \leq s < (j+1)s_p, \\ 4 \text{ rad/s} & \text{if } (j+1)s_p \leq s < (j+2)s_p, \end{cases}$$



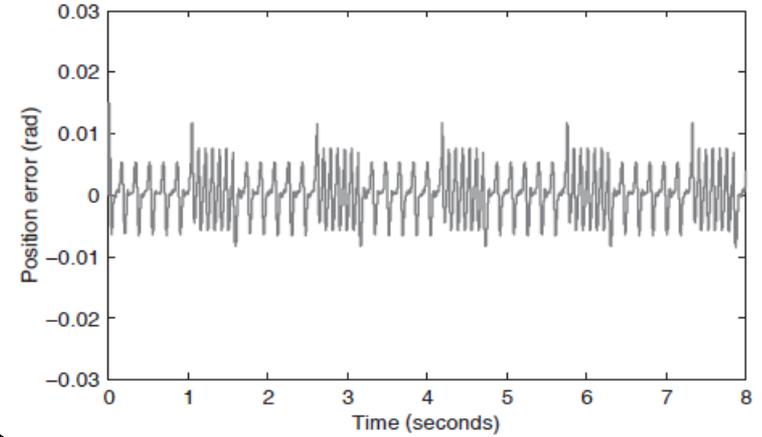
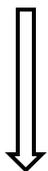
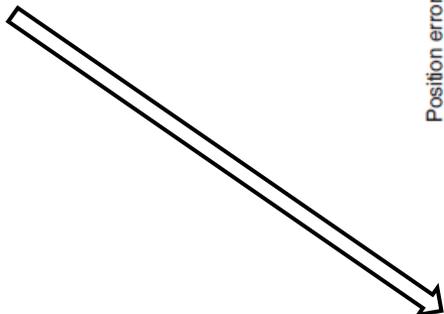
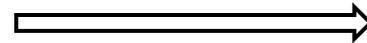
(a) Position



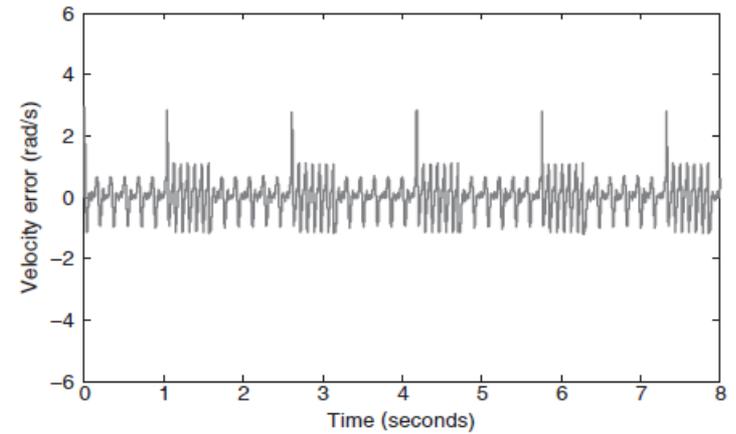
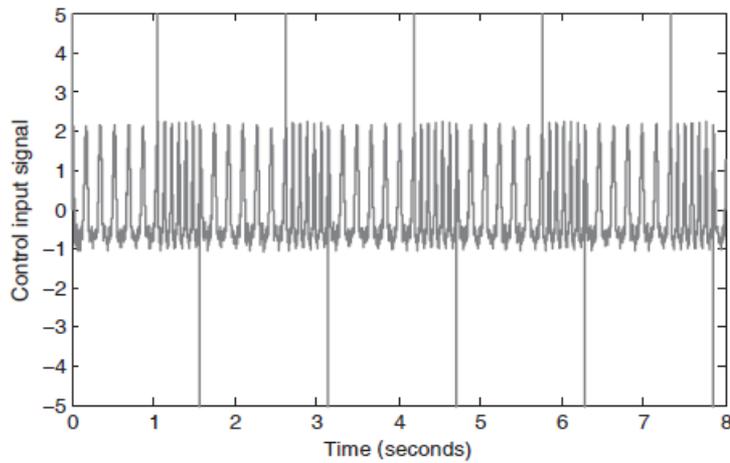
(b) Velocity

Figure: varying reference speed tracking errors without compensation

IO AC position error
 IO AC velocity error
 IO AC control signal



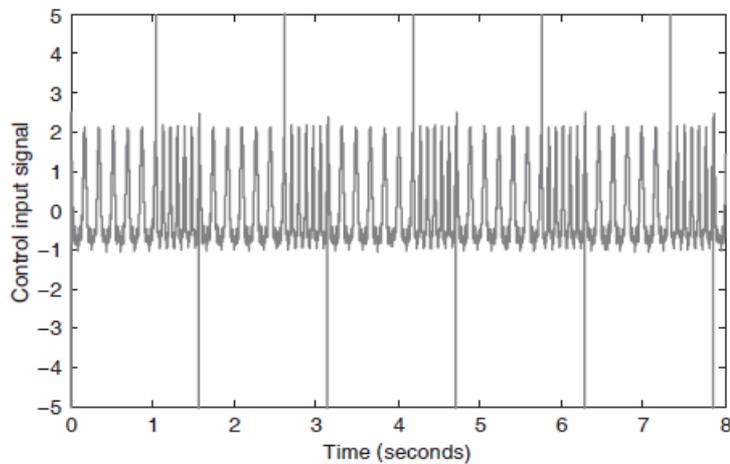
(a) Position



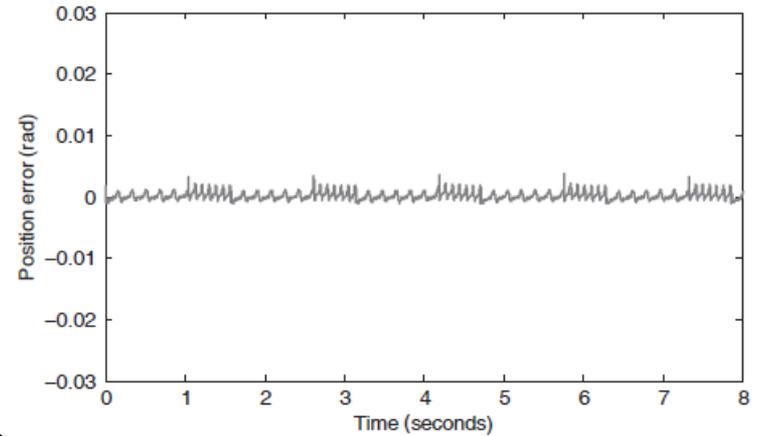
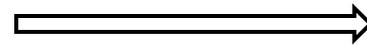
(b) Velocity

Simulation. Varying reference speed tracking control input signal with

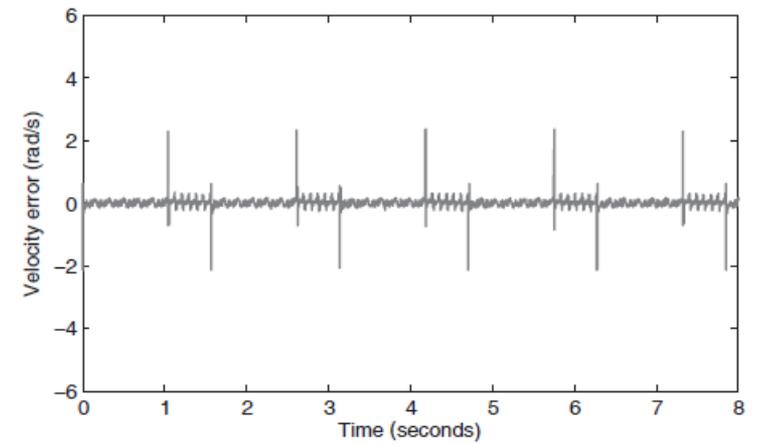
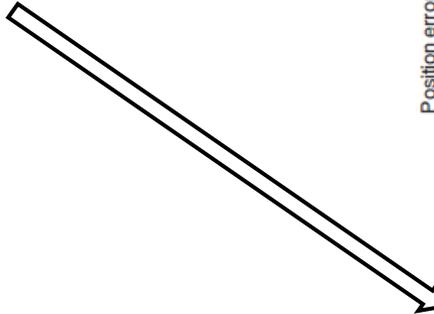
FO AC position error
FO AC velocity error
FO AC control signal



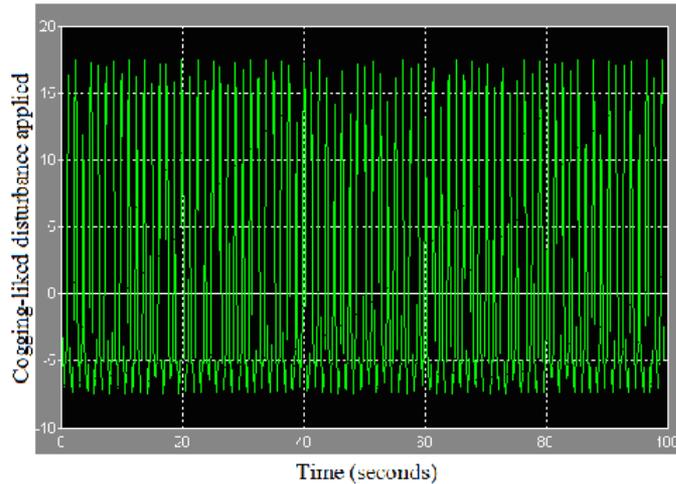
Simulation. Varying reference speed tracking control input signal



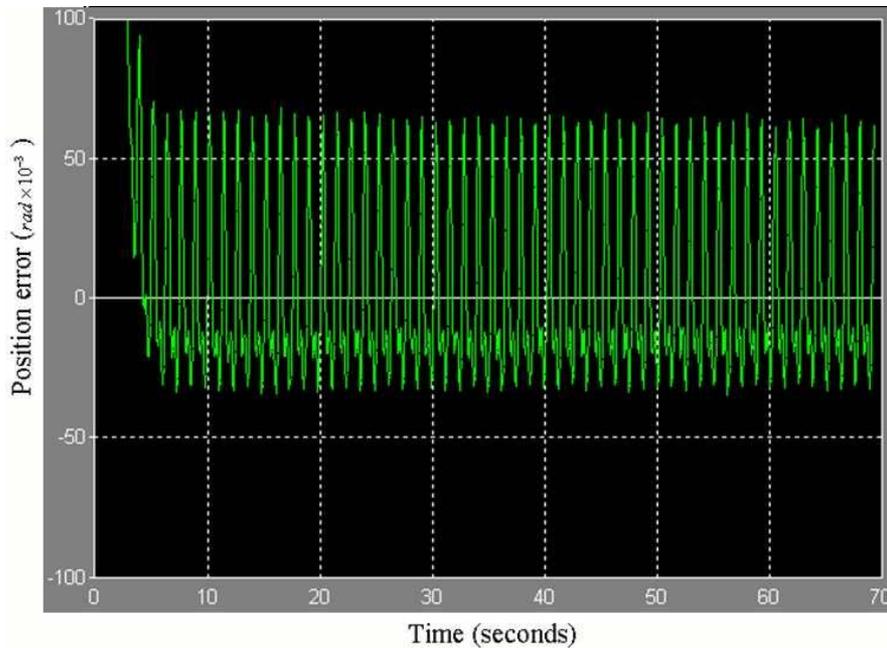
(a) Position



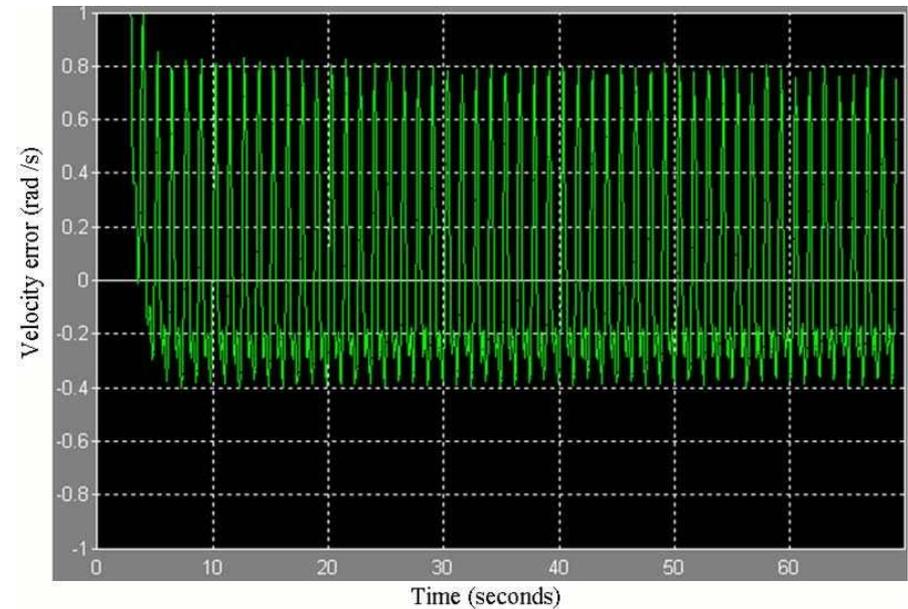
(b) Velocity

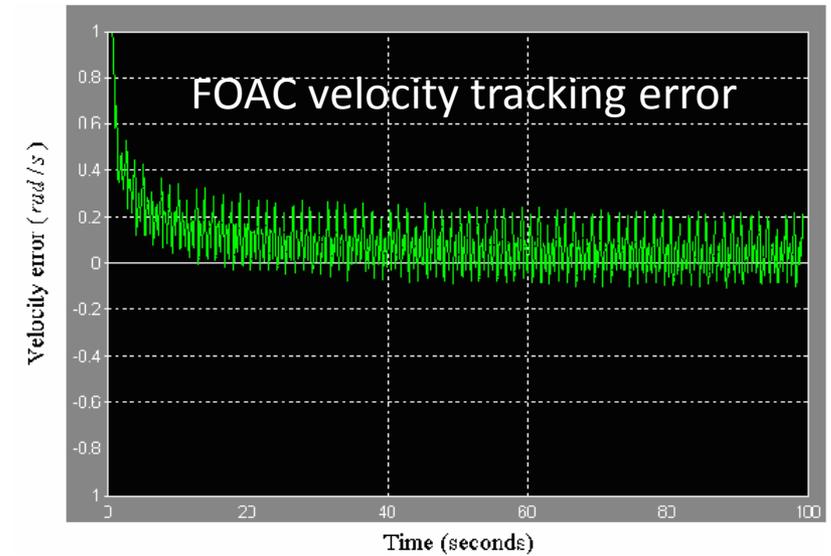
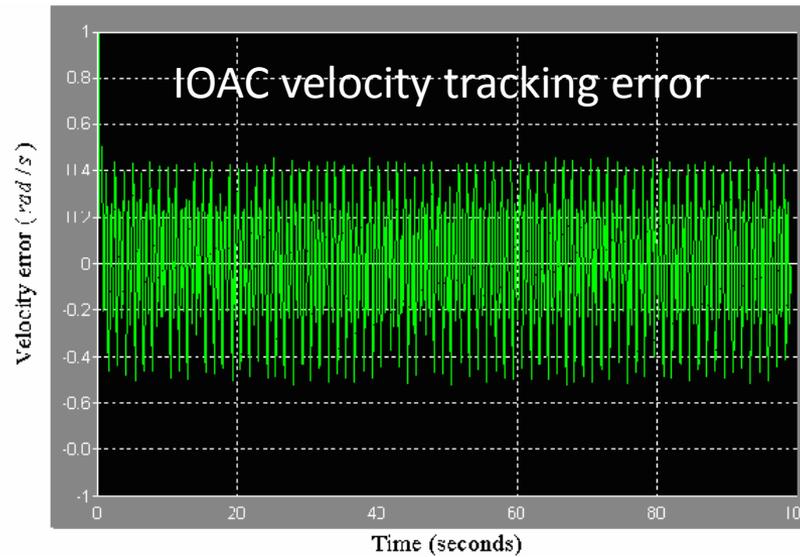
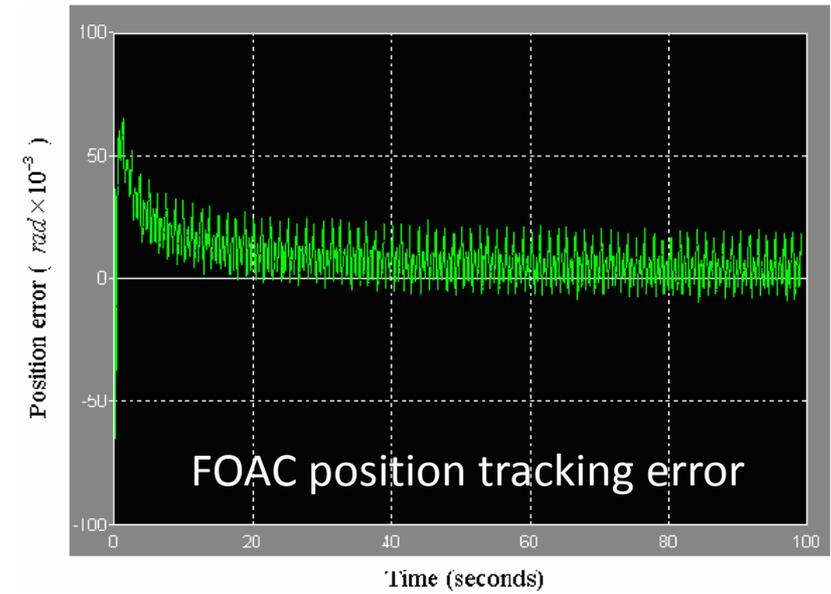
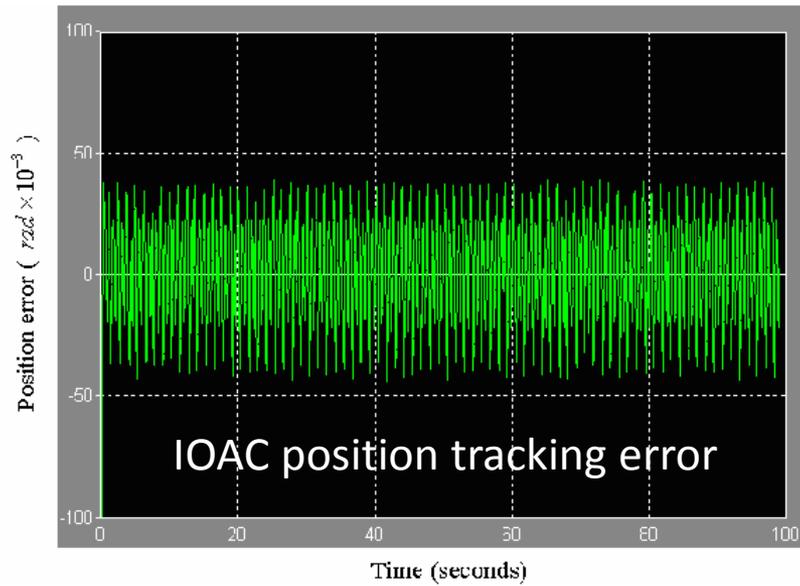


The applied cogging-like disturbance



Constant reference speed tracking errors without compensation





- PART I FUNDAMENTALS OF FRACTIONAL CONTROLS
- 1 Introduction 3

- PART II FRACTIONAL ORDER VELOCITY SERVO
- 2 Fractional Order PI Controller Designs for Velocity Servo Systems 25
- 3 Tuning Fractional Order PI Controllers for Fractional Order Velocity Systems with Experimental Validation 41
- 4 Relay Feedback Tuning of Robust PID Controllers 59
- 5 Auto-Tuning of Fractional Order Controllers with ISO-Damping 73

- PART III FRACTIONAL ORDER POSITION SERVO
- 6 Fractional Order PD Controller Tuning for Position Systems 91
- 7 Fractional Order [PD] Controller Synthesis for Position Servo Systems 105
- 8 Time-Constant Robust Analysis and Design of Fractional Order [PD] Controller 123
- 9 Experimental Study of Fractional Order PD Controller Synthesis for Fractional Order Position Servo Systems 139
- 10 Fractional Order [PD] Controller Design and Comparison for Fractional Order Position Servo Systems 155

- PART IV STABILITY AND FEASIBILITY FOR FOPID DESIGN
- 11 Stability and Design Feasibility of Robust PID Controllers for FOPTD Systems 165
- 12 Stability and Design Feasibility of Robust FOPI Controllers for FOPTD Systems 187

- **PART V FRACTIONAL ORDER DISTURBANCE COMPENSATORS**
- **13 Fractional Order Disturbance Observer 211**
- **14 Fractional Order Adaptive Feed-forward Cancellation 223**
- **15 Fractional Order Robust Control for Cogging Effect 243**
- **16 Fractional Order Periodic Adaptive Learning Compensation 275**

- PART VI EFFECTS OF FRACTIONAL ORDER CONTROLS ON NONLINEARITIES
- 17 Fractional Order PID Control of A DC-Motor with Elastic Shaft 293
- 18 Fractional Order Ultra Low-Speed Position Servo 313
- 19 Optimized Fractional Order Conditional Integrator 329

- PART VII FRACTIONAL ORDER CONTROL APPLICATIONS
- 20 Lateral Directional Fractional Order Control of A Small Fixed-Wing UAV 345
- 21 Fractional Order PD Controller Synthesis and Implementation for HDD Servo System 369

The state dependent periodic disturbance (SDPD)

Cogging force
Friction force
Etc.

The general form of SDPD

$$F_{disturbance} = \sum_{i=1}^{\infty} A_i \sin(\omega_i x + \varphi_i)$$

The plant model

$$\dot{\theta}(t) = v(t),$$

$$\dot{v}(t) = u - \frac{a(\theta)}{J} - T_l - B_1 v,$$

$$u = \frac{1}{J} T_m, T_l = \frac{1}{J} T_l, B_1 = \frac{B}{J},$$

θ : angular position, v : velocity; $a(\theta)$ unknown position-dependent cogging disturbance; J : moment of inertia; T_m : electromagnetic torque; T_l : load torque; B : viscous friction coefficient

The adaptive controller

$$u(t) = \dot{v}_d(t) + T_I + \frac{\hat{a}(t)}{J} + \alpha m(t) + \gamma e_v(t),$$

where

$$m(t) := \gamma e_\theta(t) + e_v(t),$$

The adaptive law **in and after** the first trajectory period

$$\hat{a}(t) = \begin{cases} z - \mu v, & \text{if } s < s_p & \text{FO AC} \\ \hat{a}_1(t) + \frac{K}{J} m_1(t), & \text{if } s \geq s_p & \text{FO PALC} \end{cases}$$

$${}_0D_t^\nu z(t) = \mu[\dot{v}_d(t) + \alpha m(t) + \gamma e_v(t)] + \frac{e_v(t)}{J},$$

$$\begin{cases} v = 1, & \text{IO AC + PALC} \\ v \in (0, 1), & \text{FO AC + PALC} \end{cases}$$

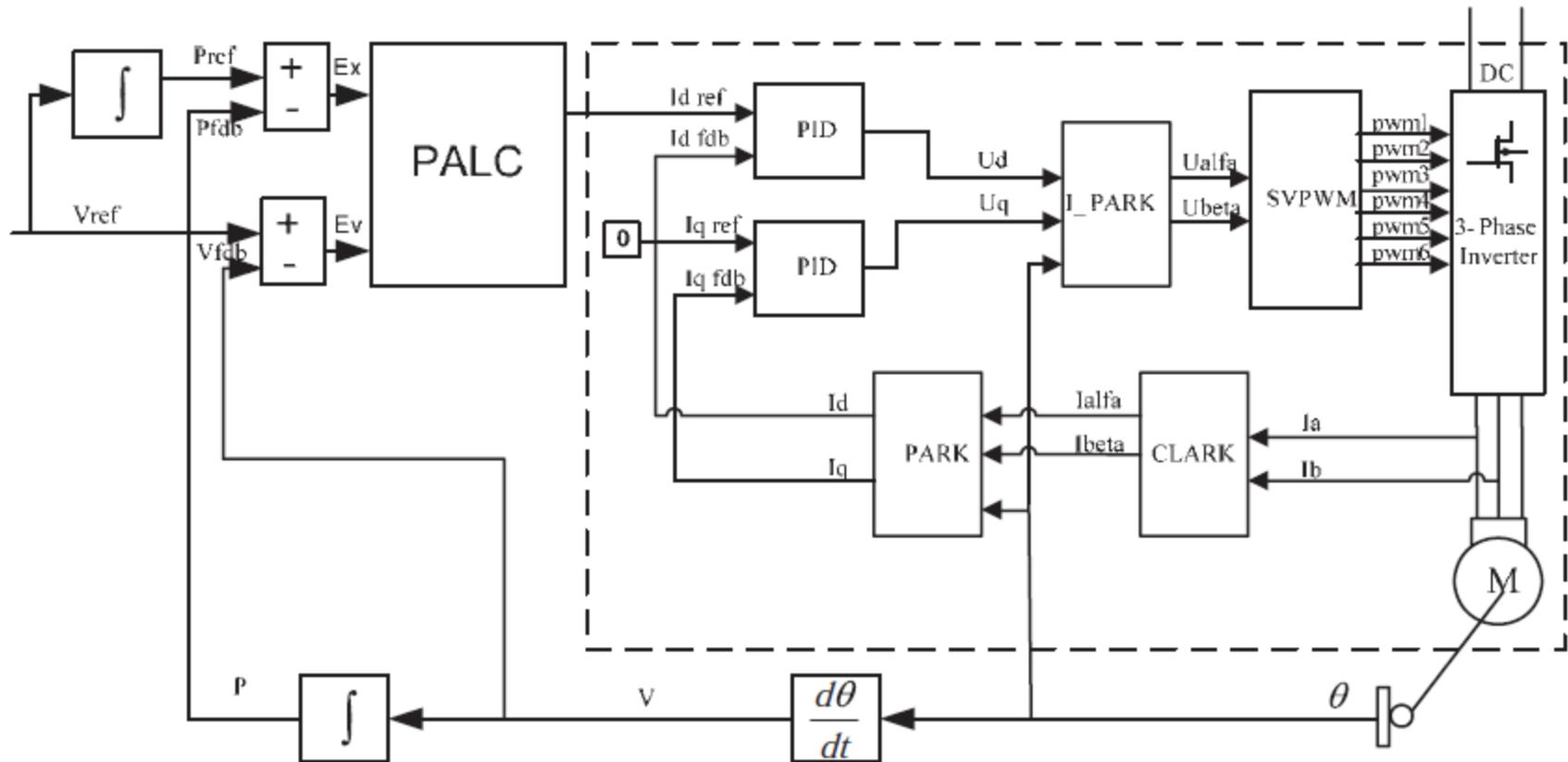


Figure: Block diagram of the cogging PALC in the PMSM position servo system model

Figure: PMSM Specifications

Rated power	1.64 Kw	Rated speed	2000 rpm
Rated torque	8 Nm	Stator resistance	2.125 Ω
Stator inductance	11.6 mH	Magnetic flux	0.387 Wb
Number of poles	6	Moment of inertia	0.00289 kgm ²
Friction coefficient	0.0003 Nms		

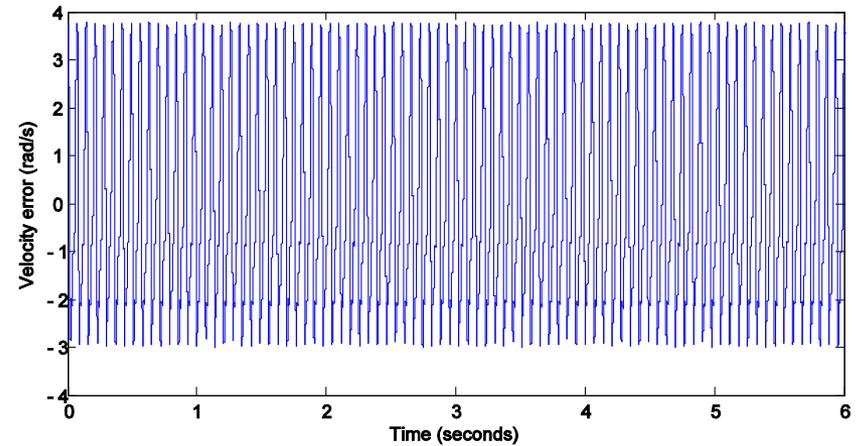
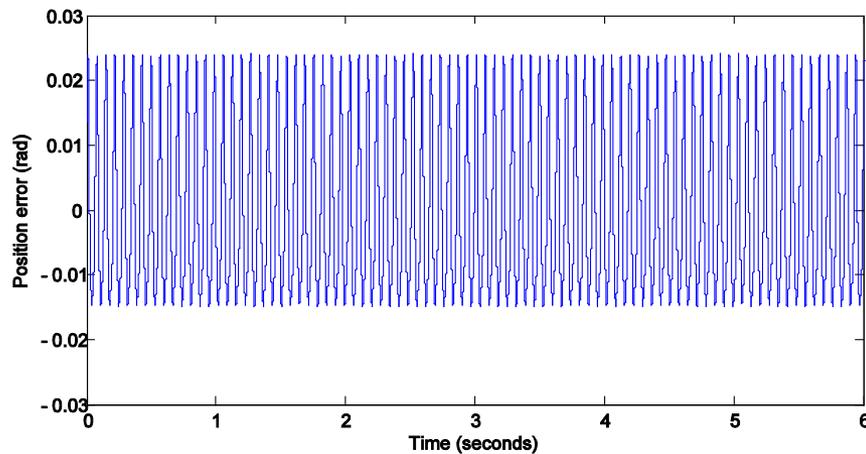
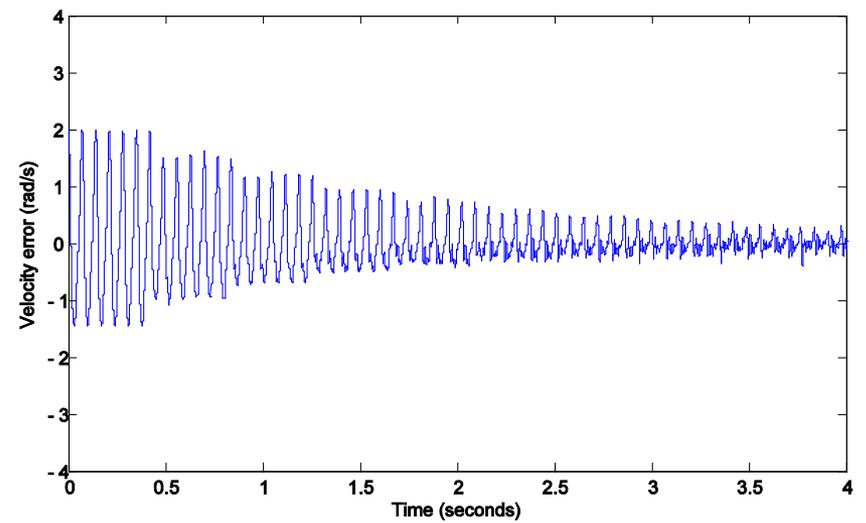
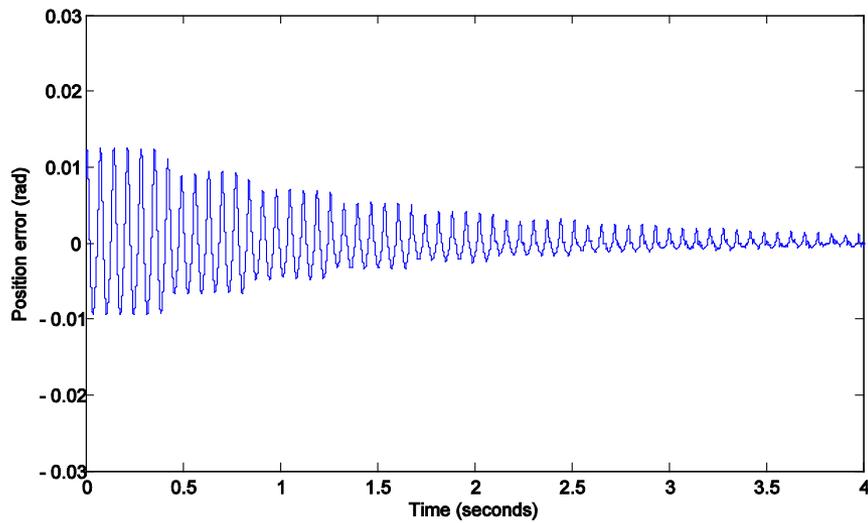
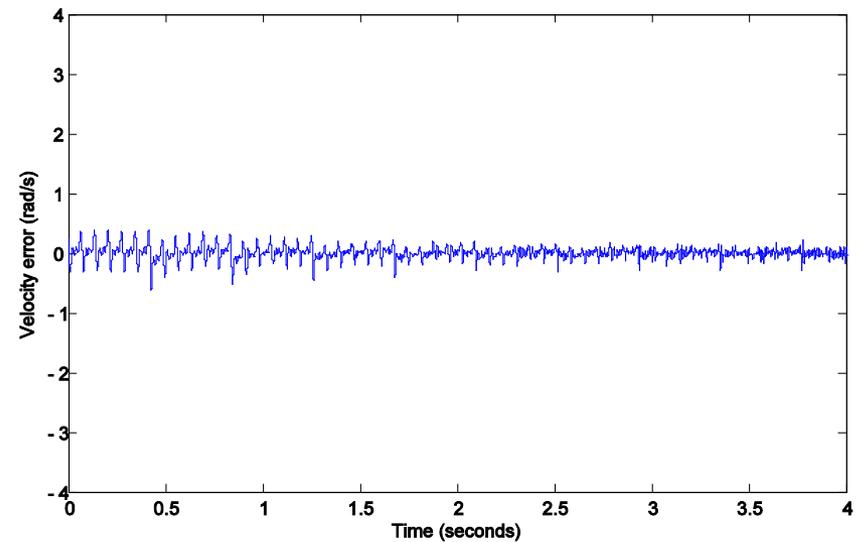
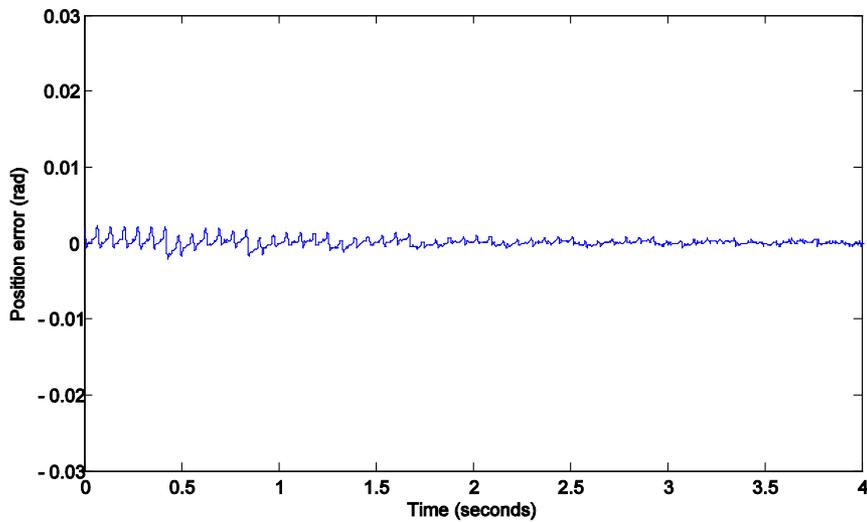
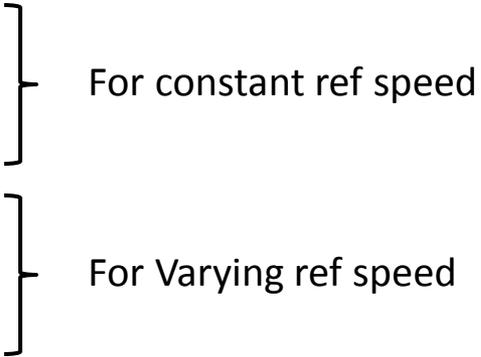


Figure: Tracking errors without any compensation



FO AC+PALC (above)
IO AC+PALC (below)



- FO DOB
 - No DOB
 - IO DOB
 - FO DOB
- FO AFC
- FO AC
 - No compensation
 - IO AC
 - FO AC
 - No compensation
 - IO AC
 - FO AC
- FO AC+PALC
 - No compensation
 - FO AC+PALC
 - IO AC+PALC

This is the end of session V

Questions?