Turkish National Meeting on Automatic Control
(TOK 2013) , Sept. 25, 2013, Malatya, Turkey

A Tutorial on Fractional Order Motion Control

Part II: Fractional Order Velocity Servo

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- Fractional Order Motion Controls

John Wiley & Sons, Inc.

Hardcover, 454 pages, December 2012

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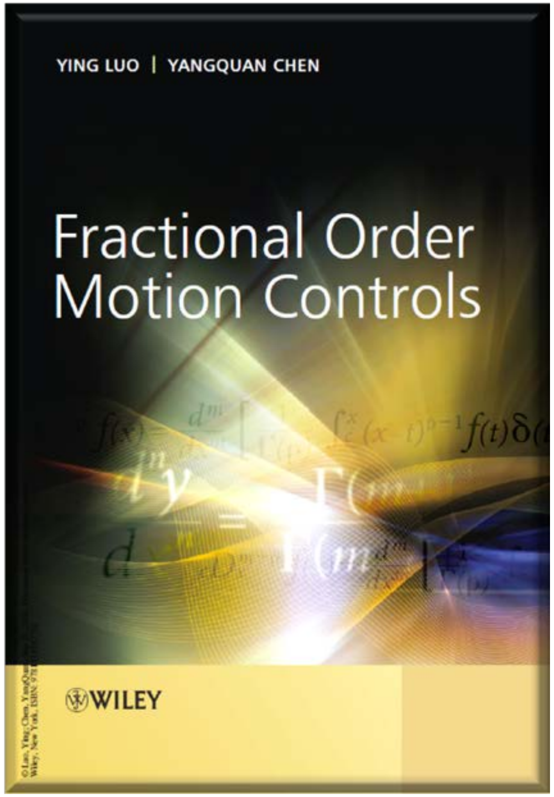


Scope of the book

IO model, FO controller

The simplified velocity Servo Systems:
First order plus time delay (FOPTD) systems:

$$P(s) = \frac{K}{Ts+1}e^{-Ls} \tag{12}$$

Three controllers

- Integer order PID

$$C_1(s) = K_p + \frac{K_i}{s} + K_d s, \tag{13}$$

- Fractional order PI

$$C_2(s) = K_p(1 + \frac{K_i}{s^\lambda}), \tag{14}$$

- Fractional order [PI]

$$C_3(s) = (K_p + \frac{K_i}{s})^\lambda, \tag{15}$$

where, $\lambda \in (0,2)$ and $\gamma \in (0,2)$.

The “flat phase” tuning rules

- Phase margin

$$\angle[G(j\omega_c)] = Arg[C(j\omega_c)P(j\omega_c)] = -\pi + \phi_m$$

- Gain crossover frequency

$$|G(j\omega_c)| = |C(j\omega_c)P(j\omega_c)| = 0$$

- Robustness: “flat phase”

$$\left.\frac{d(\angle G(j\omega_c))}{d\omega}\right|_{\omega=\omega_c} = 0$$

Benefits:

Robust to gain variations

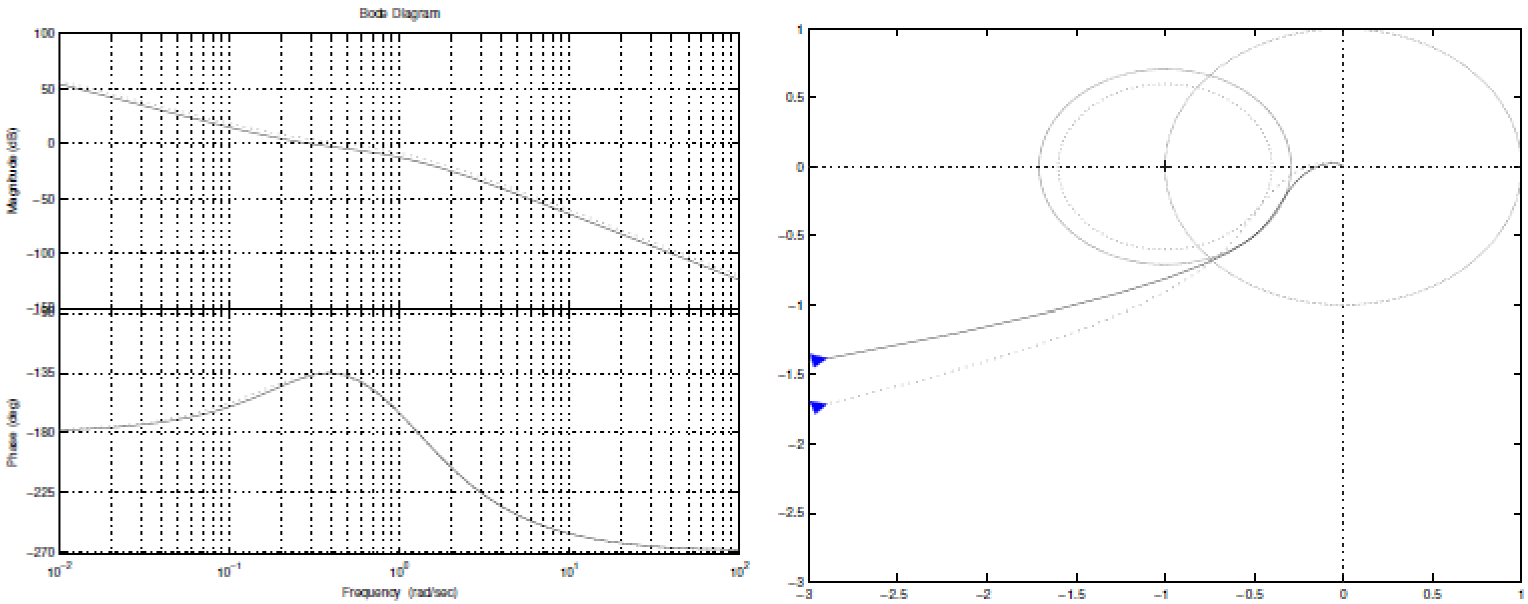
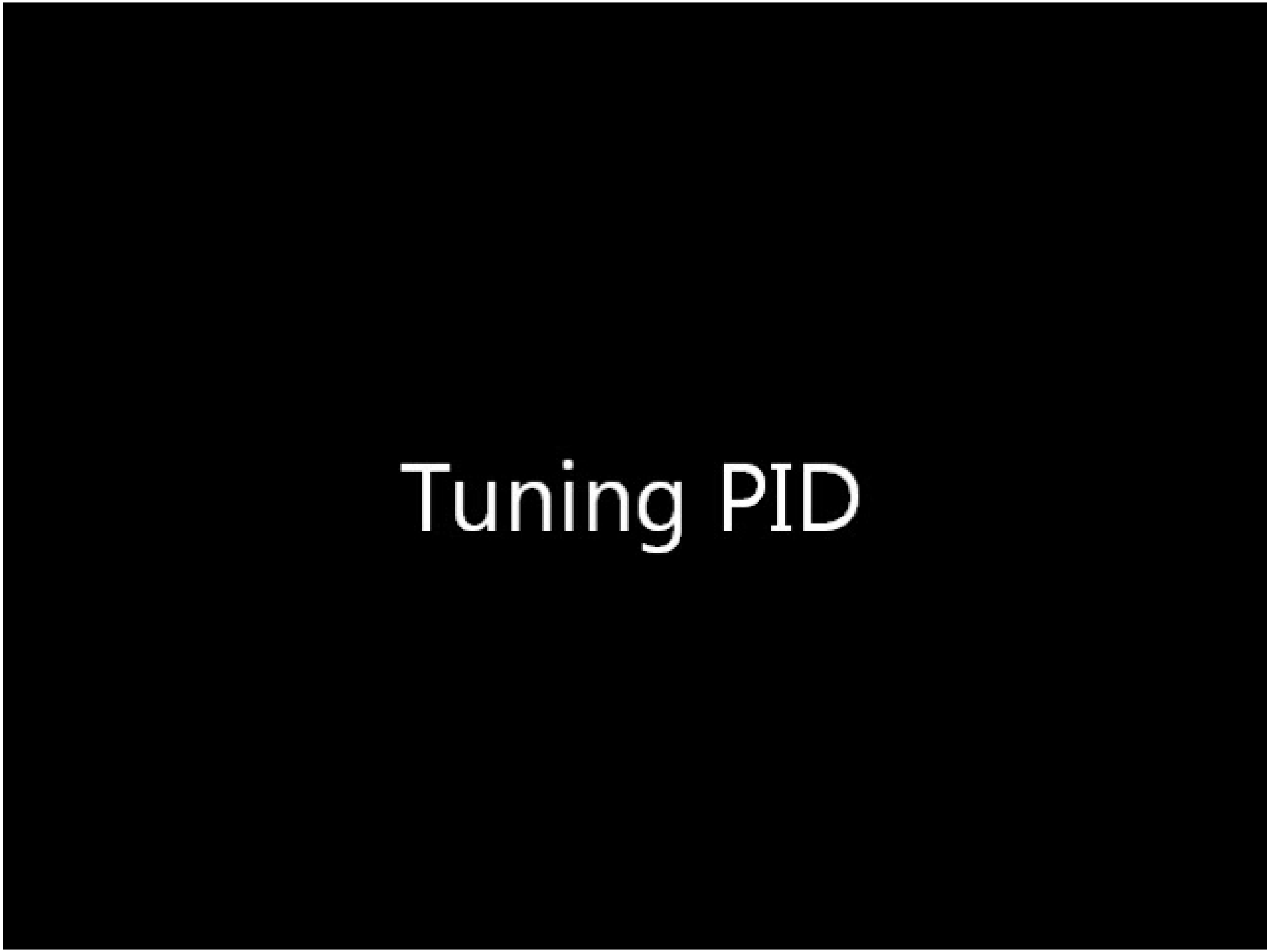


Figure: An illustrative view of the “flat phase” concept

A video illustration



- The Iso-damping property

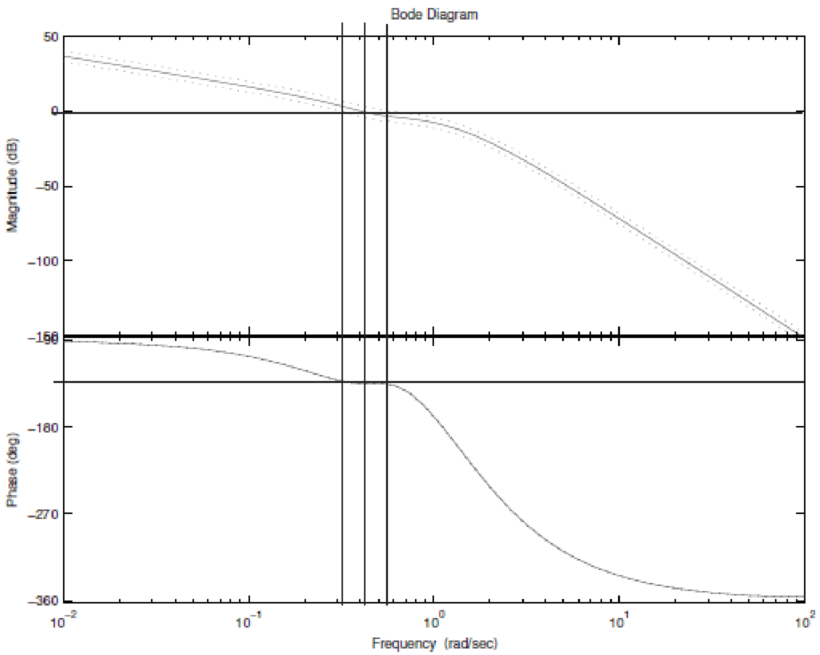
$$\frac{d\angle G(s)}{ds}|_{s=j\omega_c} = 0$$

- Bode's ideal transfer function

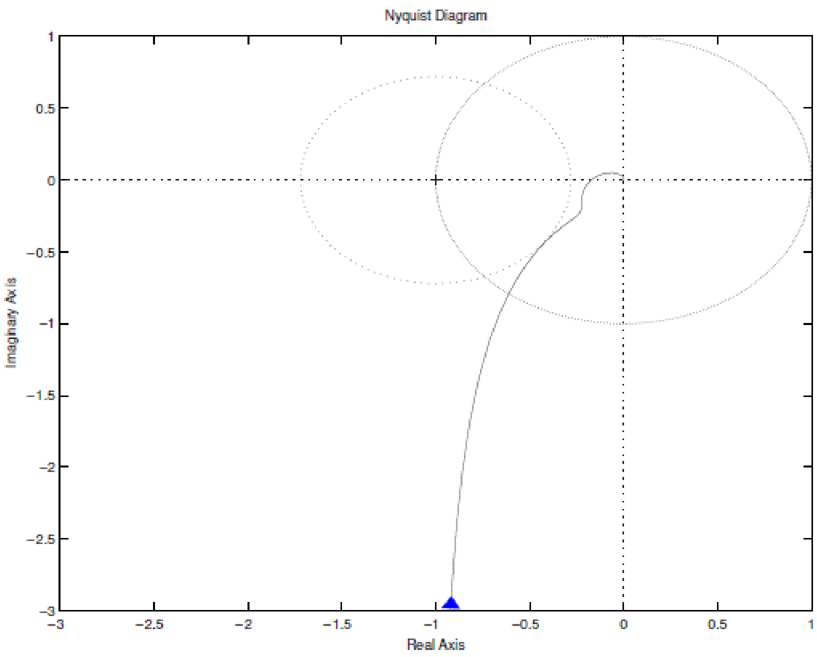
$$L(s) = \left(\frac{s}{\omega_{gc}}\right)^{\alpha}$$

H. W. Bode. *Network Analysis and Feedback Amplifier Design*. New York, Van Nostrand, 1945

YQ. Chen, CH. Hu, and K. L. Moore, "*Relay Feedback Tuning of Robust PID Controllers With Iso-Damping Property*", Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, Vol. 35. Issue: 1. 2005.



(a) Basic idea: a flat phase curve at gain crossover frequency



(b) Sensitivity circle tangentially touches Nyquist curve at the flat phase

Frequency response of the FO PI controller

$$j^{-\lambda} \triangleq e^{-\frac{\pi}{2}j\lambda} \quad \Rightarrow$$

$$\begin{aligned} C_2(j\omega) &= K_p(1 + K_i(j\omega)^{-\lambda}) && (16) \\ &= K_p(1 + K_i\omega^{-\lambda}\cos(\lambda\frac{\pi}{2}) - jK_i\omega^{-\lambda}\sin(\lambda\frac{\pi}{2})) && (17) \end{aligned}$$

Gain

$$\begin{aligned} |G_2(j\omega)| &= |C_2(j\omega)||P(j\omega)| \\ &= \frac{K_p\sqrt{[1+K_i\omega^{-\lambda}\cos(\lambda\pi/2)]^2+K_i\omega^{-\lambda}\sin(\lambda\pi/2)^2}}{\sqrt{1+(\omega T)^2}}. \end{aligned} \quad (18)$$

Phase

$$\begin{aligned} \angle[G_2(j\omega)] &= \angle[C_2(j\omega)]+\angle[P(j\omega)] \\ &= -\tan^{-1}[\frac{K_i\omega^{-\lambda}\sin(\lambda\pi/2)}{1+K_i\omega^{-\lambda}\cos(\lambda\pi/2)}]-\tan^{-1}(\omega T)-L\omega, \end{aligned} \quad (19)$$

Flat phase

$$\frac{d(\angle(G_2(j\omega)))}{d\omega}|_{\omega=\omega_c} = \frac{K_i\lambda\omega_c^{\lambda-1}\sin(\lambda\pi/2)}{\omega_c^{2\lambda}+2K_i\omega_c^{\lambda}\cos(\lambda\pi/2)+K_i^2}-E_2=0. \quad (20)$$

People who are using this

2013 American Control Conference (ACC)
Washington, DC, USA, June 17-19, 2013

Design of a Fractional – Order Controller for the Setpoint Ramp Tracking Problem

A. Morell[†], J.J. Trujillo[*], M. Rivero[§] and L. Acosta[†] (*IEEE Member*)

2) Robustness against gain variations:

$$\left.\frac{\mathrm{d}\left(\arg\left[C(s)G(s)\right]\right)}{\mathrm{d}w}\right|_{w=w_{cg}} = 0, \qquad (22)$$

Fractional-order control of a Steward platform by means of numerical and genetic algorithm optimization

In order to design a fractional-order PD^{u} controller, three specifications are proposed from the basic definition of gain crossover frequency and phase margin [22]:

1. Phase margin specification:

$$\arg[H(j\omega_{cg})] = -\pi + \varphi_m. \qquad (28)$$

2. Robustness to variation in the gain of the plant, which is the derivative of the phase if the open-loop system with respect to the frequency is forced to be zero at the gain crossover frequency so that the closed-loop system is robust to gain variations, and therefore the overshoots of the response are almost invariant.

$$\left.\frac{d\left(\arg\left(H(j\omega_{cg})\right)\right)}{d\omega}\right|_{\omega=\omega_{cg}} = 0. \qquad (29)$$

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Identification and tuning fractional order proportional integral controllers for time delayed systems with a fractional pole

Hadi Malek*, Ying Luo, YangQuan Chen
Electrical & Computer Eng. Dept., Utah State University, Logan, UT 84321, United States

Also works for process control !

2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Fractional Order PID Controller (FOPID)-Toolbox

Nabil Lachhab[1], Ferdinand Svaricek[1], Frank Wobbe[2] and Heiko Rabba[2]

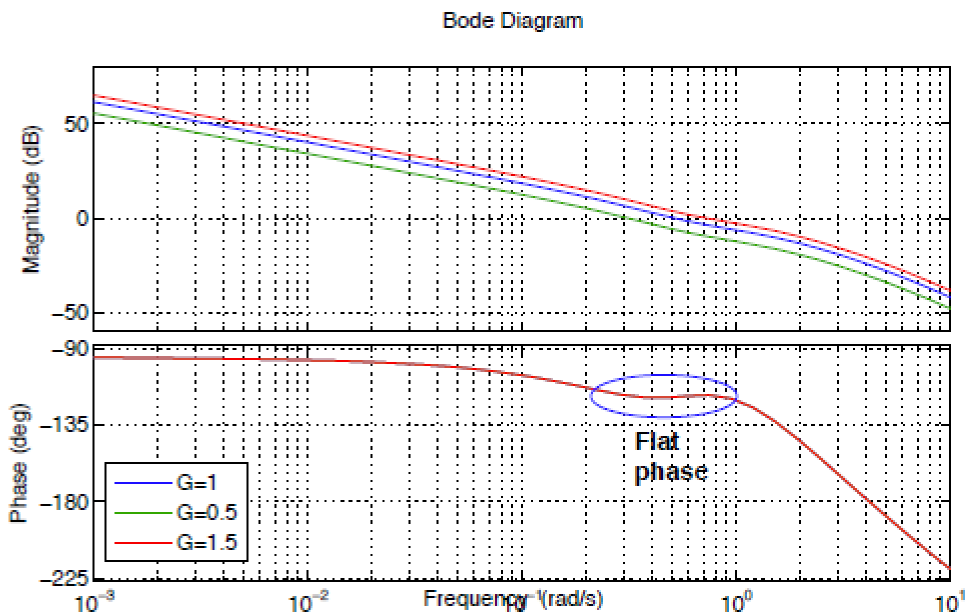


Fig. 11. Open-loop bode plot $PI^{\alpha}D^{\beta}$

Frequency response of the FO [PI] controller

$$\begin{aligned} C_3(j\omega) &= (K_p + (\frac{K_i}{j\omega}))^\lambda \\ &= [K_p^2 + \frac{K_i}{\omega}^2]^{\lambda/2} e^{-j\lambda \arctan(K_i/(K_p\omega))} \end{aligned} \tag{21}$$

Phase and gain

$$\begin{aligned} \angle[G_3(j\omega)] &= \angle[C_3(j\omega)] + \angle[P(j\omega)] \\ &= -\lambda \tan^{-1}(\frac{K_i}{K_p\omega}) - \tan^{-1}(T\omega) - L\omega, \end{aligned} \tag{22}$$

$$\begin{aligned} |G_3(j\omega)| &= |C_3(j\omega)||P(j\omega)| \\ &= \frac{[K_p^2 + (K_i/\omega)^2]^{\lambda/2}}{\sqrt{1 + (T\omega)^2}}. \end{aligned} \tag{23}$$

Scope of the book

Fractional order velocity system

A first order system:

$$P(s) = \frac{K}{Ts^{\alpha} + 1} \tag{24}$$

Fractional diffusion modeling of ion channel gating

Igor Goychuk* and Peter Hänggi

Institute of Physics, University of Augsburg, Universitätsstr. 1, D-86135 Augsburg, Germany

(Dated: February 2, 2008)

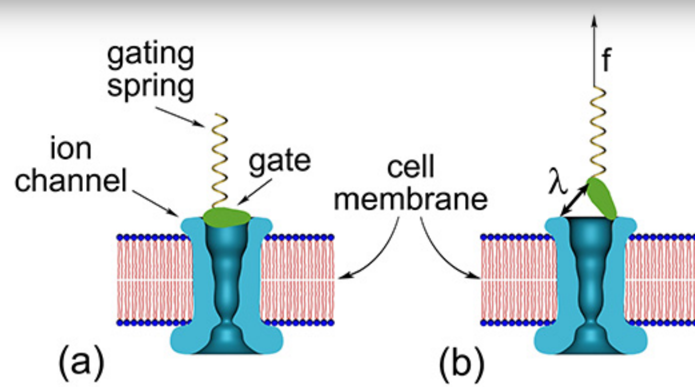


Figure: ion channel gating

Source: http://www.quantbiolab.com/research-interests/sensory-systems-nature-and-laboratory/mechanical-magnetoreception-animals

Tuning specifications

The “flat phase” concept

Analytic solution

For FOPI,

$$K_i = \frac{-(\lambda \sin(\frac{\lambda\pi}{2}) + 2\cos(\frac{\lambda\pi}{2}) s_p(\omega_c)) + \sqrt{\triangle}}{2\omega_c^{-\lambda} s_p(\omega_c)} \tag{31}$$

$$K_p = \frac{\cos(\phi_m)}{|p(j\omega_c)|\sqrt{1 + 2K_i\omega_c^{-\lambda}\cos(\lambda\pi/2) + (K_i\omega_c^{-\lambda})^2}}. \tag{32}$$

where

$$\triangle = \lambda^2 \sin^2(\frac{\lambda\pi}{2}) + 2\lambda\sin(\lambda\pi)s_p(\omega_c) - 4\sin^2(\frac{\lambda\pi}{2})s_p^2(\omega_c). \tag{33}$$

The approximation of $s_p(\omega_c)$ for unknown stable plant can be given as:

$$s_p(\omega_c) \approx \angle P(j\omega_c) + \frac{2}{\pi}[\ln|K_g| - \ln|P(j\omega_c)|],$$

Analytic solution

For FO[PI], the frequency response are

$$\angle[C_2(j\omega)] = -\lambda \arctan(\frac{K_i}{\omega}), \quad (34)$$

$$|C_2(j\omega)| = K_p(\sqrt{1+\frac{K_i^2}{\omega^2}})^\lambda. \quad (35)$$

The solution is $K_i = \omega_c \tan(\varphi)$, where, $\varphi = (\pi + \angle P(j\omega_c) - \phi_m)/\lambda$.

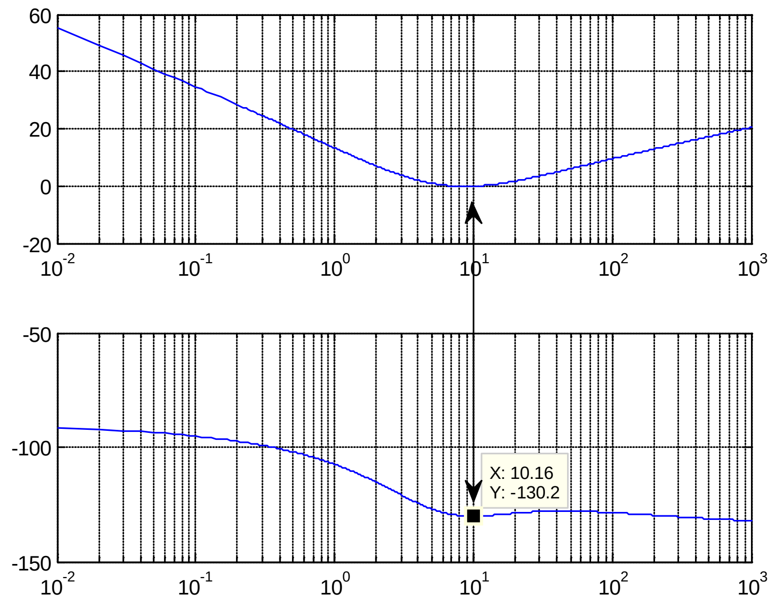
$$K_p = \frac{\cos(\phi_m)}{|P(j\omega_c)|(\sqrt{1+K_i^2/\omega_c^2})^\lambda}. \quad (36)$$

where $s_p(\omega_c)$ can be approximated in the same way as for FOPI.

- Integer order PID controller design

  (1) Given K=1, T=0.4 s, ω_c =10 rad/s, ϕ_m =50°, and α =0.5.
- Resulted controller parameters

$$Ki = 5.9319,\ Kd = -0.1433,\ Kp = -0.5325$$



Unstable

Figure: closed loop Bode plot with the designed integer order PID controller

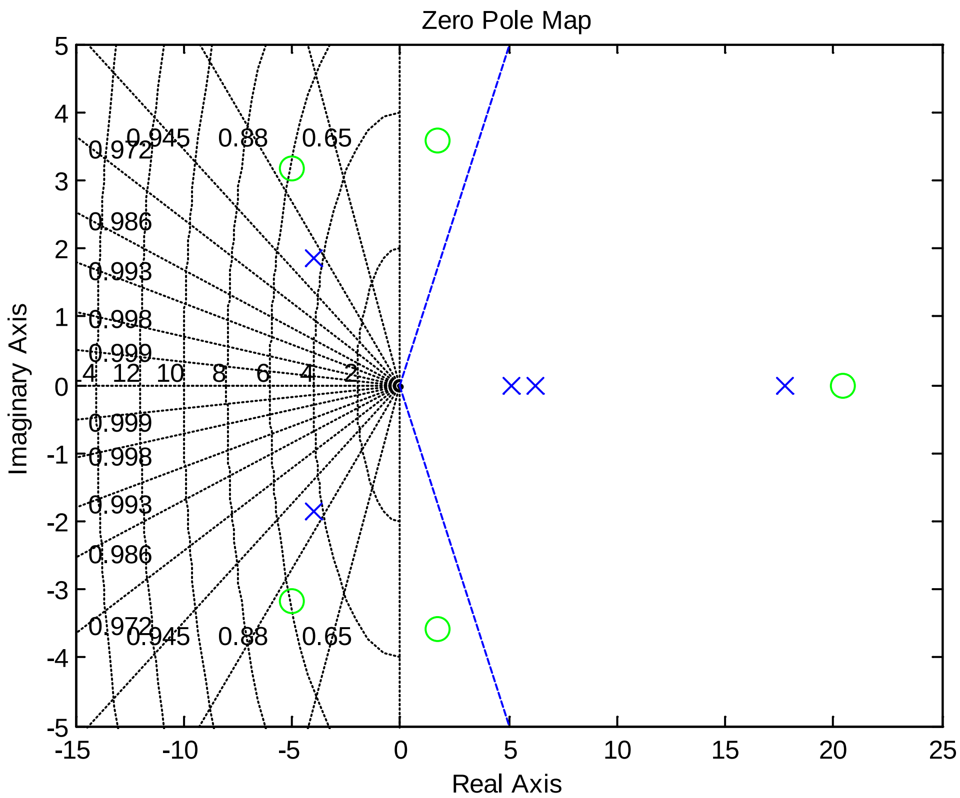
P = 1 / (0.4s^{0.5}+1)

C = kp+ki/s+kd*s =

-0.14s^{2}-0.53s+5.93
----
s

Poles:

17.7872
-4.0212 + 1.8619i
-4.0212 - 1.8619i
6.2277
5.1551



$$G = \frac{-0.056x^{2.5} + 0.14x^2 + 0.212x^{1.5} + 0.53x + 2.37x^{0.5} + 5.93}{-0.056x^{2.5} + 0.02x^2 + 0.588x^{1.5} + 0.47x + 2.37x^{0.5} + 5.93}$$

How to compute zeros and poles for such a system?

- FO characteristic equations
- Commensurate order VS Non-commensurate order

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Short communication

Root locus of fractional linear systems

J.A. Tenreiro Machado

Institute of Engineering of Porto, Dept. of Electrical Engineering, Rua Dr. António Bernardino de Almeida, 431, 4200-072 Porto, Portugal

Research Article

Extending the Root-Locus Method to Fractional-Order Systems

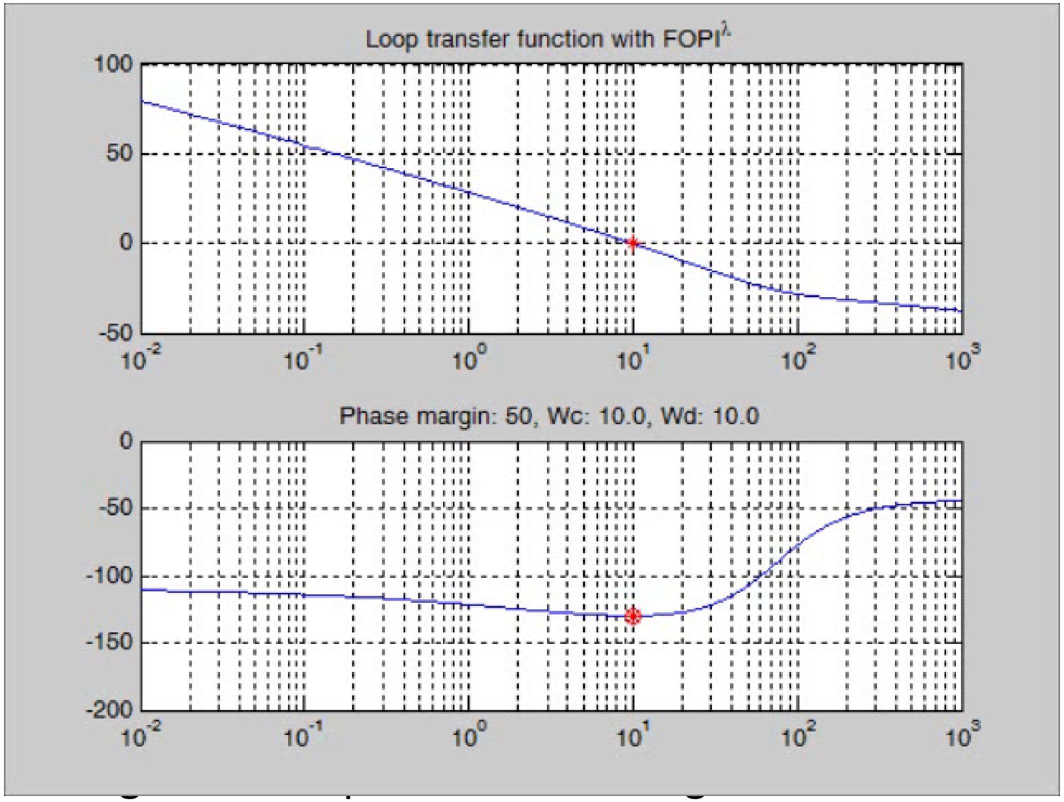
Farshad Merrikh-Bayat[1] and Mahdi Afshar[2]

[1] *Department of Electrical Engineering, Zanjan University, Zanjan, Iran*
[2] *Department of Mathematics, Zanjan Azad University, Zanjan, Iran*

- FO PI controller design

  (1) Given K=1, T=0.4 s, ω_c =10 rad/s, ϕ_m =50°, and α =0.5.
- Resulted controller parameters

$$\lambda = 1.216, \quad Ki = 194.4, \quad Kp = 0.1817$$



order PI controller

Analytical solution VS. graphic solution

- A remark for practice
  - Extremely complicated to calculate the analytical solution
  - Numerical search is usually the approach

- Two techniques for numerically solve the 3 tuning equations
  - (a). Find the intersection of the two curves

$$K_i = \frac{-B \pm \sqrt{B^2 - 4A^2\omega_c^{-2\lambda}}}{2A\omega_c^{-2\lambda}}.$$

$$K_i = \frac{-\tan[\arctan(\omega_c T) + \phi_m + L\omega_c]}{M},$$

  - (b). Align the three frequencies: ω_c, ω_d, and ω_Φ

- Dealing with
  - More than one cross over
  - More than one extreme point on phase plot

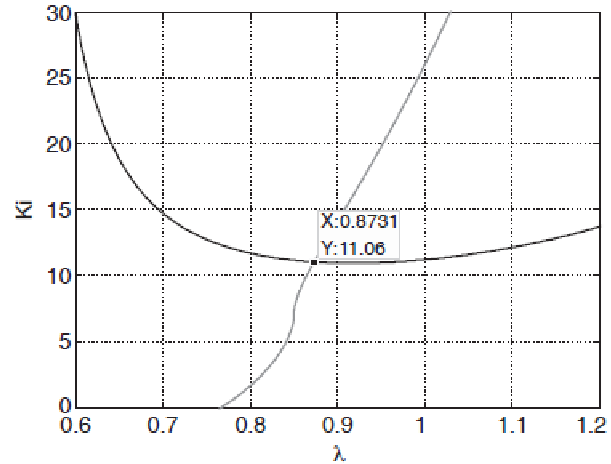


Figure: ki versus λ

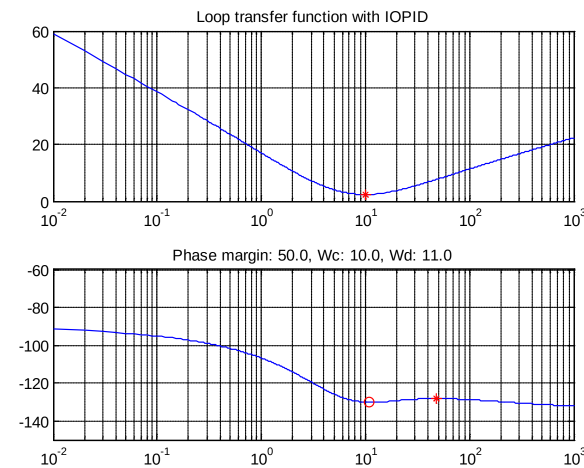


Figure: Graphically solving using Bode plot

- FO [PI] controller design

  (1) Given K=1, T=0.4 s, ω_c =10 rad/s, ϕ_m =50°, and α =0.5.
- Resulted controller parameters

$$\lambda = 1.229, \; Ki = 18.19, \; Kp = 0.1521$$

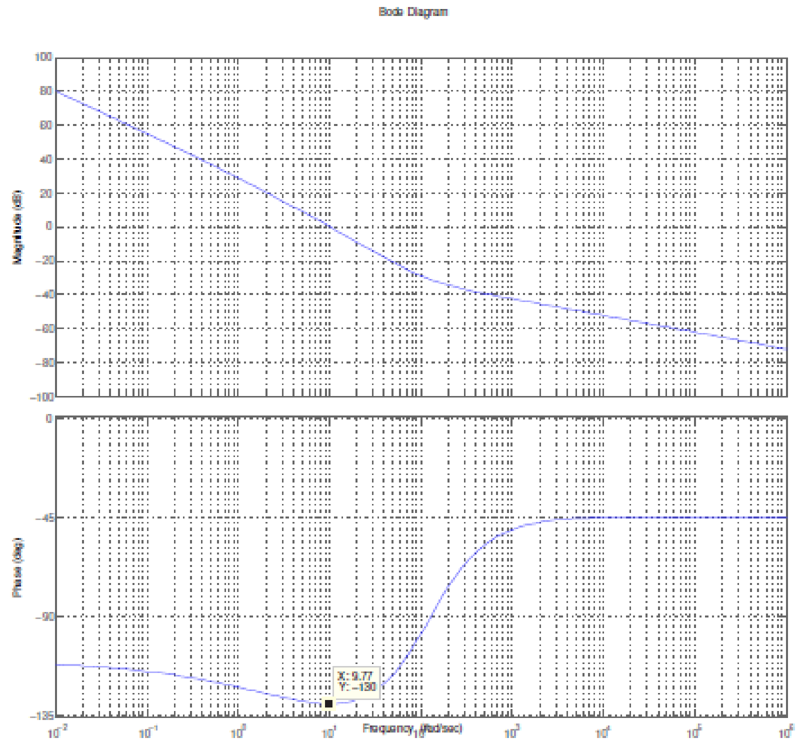
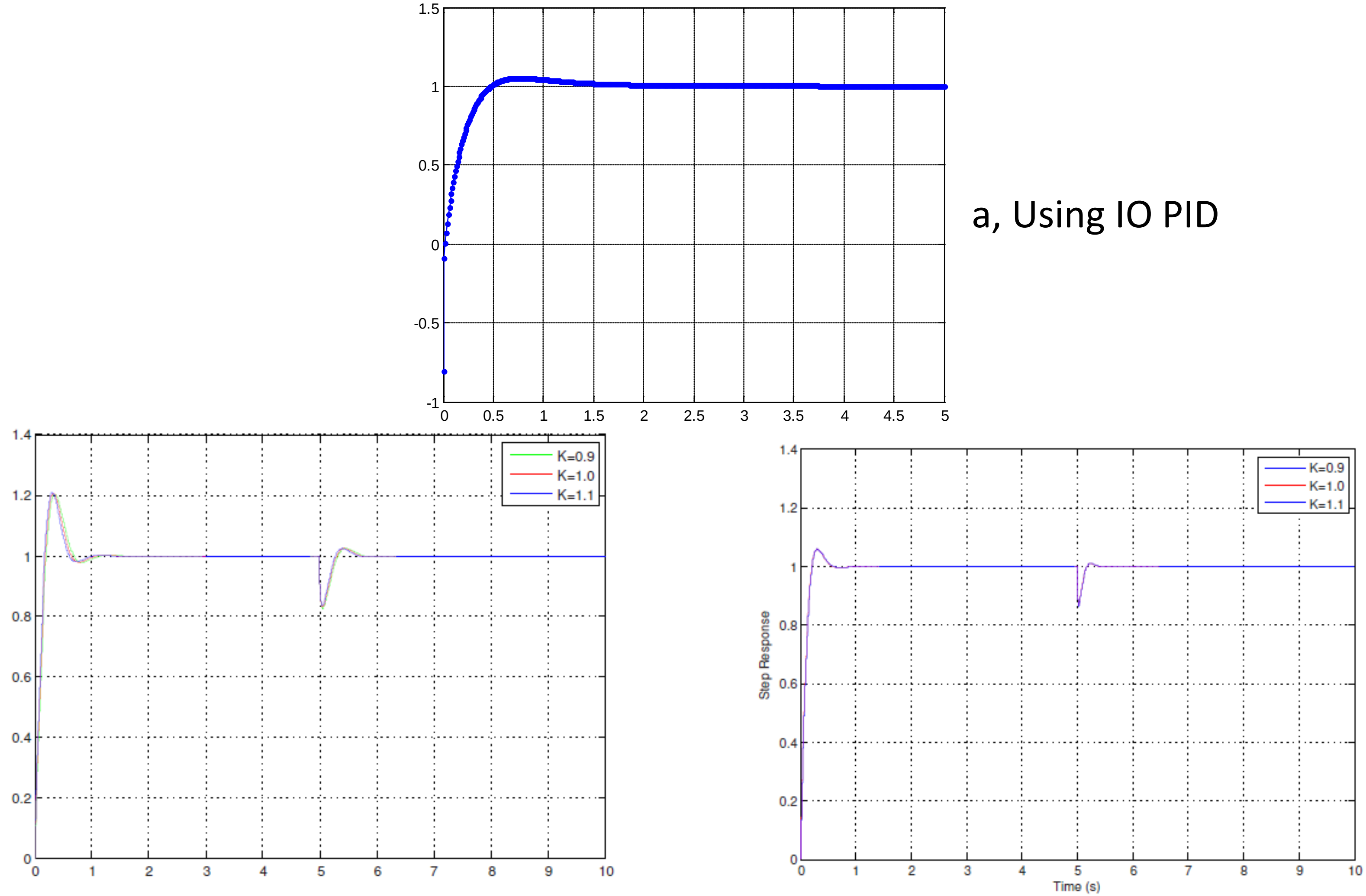


Figure: Bode plot with the designed fractional order [PI] controller



a, Using IO PID

b, Using FO PI

c, using FO [PI]

Figure: Step responses and disturbance rejections with loop gain variations

Scope of the book

- Controller form

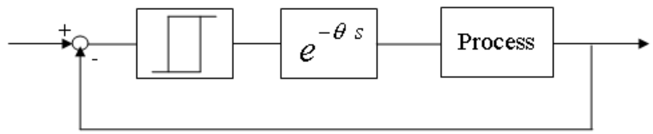
$$C(s) = K_p(1 + \frac{1}{T_i s} + T_d s).$$

- Open loop system phase

$$\angle G(s)|_{s=j\omega_c} = \Phi_m - \pi.$$

- Assume the system gain

$$|G(j\omega_c)| = \cos(\Phi_m).$$



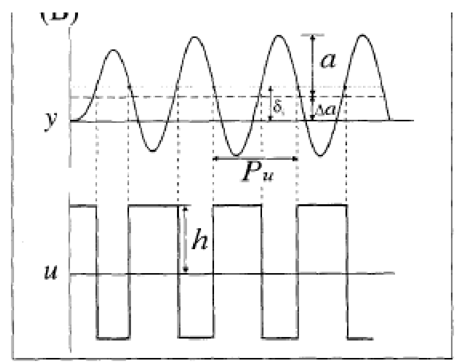
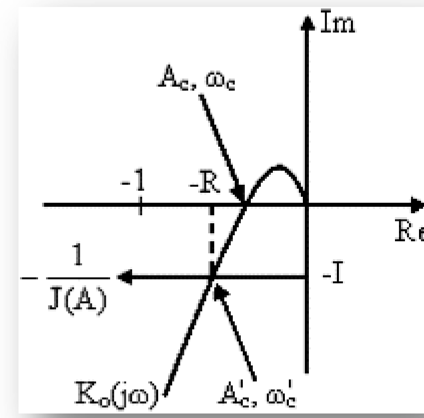


Figure 1. Input-biased relay feedback system.



A useful relationship

When phase is flat, i.e. $\frac{d\angle G(j\omega)}{d\omega}\Big|_{\omega=\omega_c} = 0$, the following equation holds,

$$\angle \frac{dG(j\omega)}{d\omega}\Big|_{\omega=\omega_c} = \angle G(j\omega)\Big|_{\omega=\omega_c}$$

The new relationship

$$T_d = \frac{-T_i\omega_0 + 2s_p(\omega_0) + \sqrt{\Delta}}{2s_p(\omega_0)\omega_0^2 T_i},$$

where $\Delta = T_i^2\omega_0^2 - 8s_p(\omega_0)T_i\omega_0 - 4T_i^2\omega_0^2 s_p^2(\omega_0).$

The new tuning formulae

$$K_p = \frac{\cos(\Phi_{\mathrm{m}})}{|P(j\omega_c)\sqrt{1+\tan^2(\Phi_m - \angle P(j\omega_c))}|},$$
$$T_i = \frac{-2}{\omega_c[s_p(\omega_c) + \hat{\Phi}) + \tan^2(\hat{\Phi})s_p(\omega_c)]},$$

where $\hat{\Phi} = \Phi_m - \angle P(j\omega_c)$

An iterative algorithm

(1) Start with the desired tangent frequency ω_c.

(2) Select two different values (θ_{-1} and θ_0) for the time delay parameter properly and do the relay feedback test twice. Then, two points on the Nyquist curve of the plant can be obtained. The frequencies of these points can be represented as ω_{-1} and ω_0 which correspond to θ_{-1} and θ_0, respectively. The iteration begins with these initial values $(\theta_{-1}, \omega_{-1})$ and (θ_0, ω_0).

(3) With the values obtained in the previous iterations, the artificial time delay parameter θ can be updated using a simple interpolation/extrapolation scheme as follows:

$$\theta_n = \frac{\omega_c - \omega_{n-1}}{\omega_{n-1} - \omega_{n-2}}(\theta_{n-1} - \theta_{n-2}) + \theta_{n-1}$$

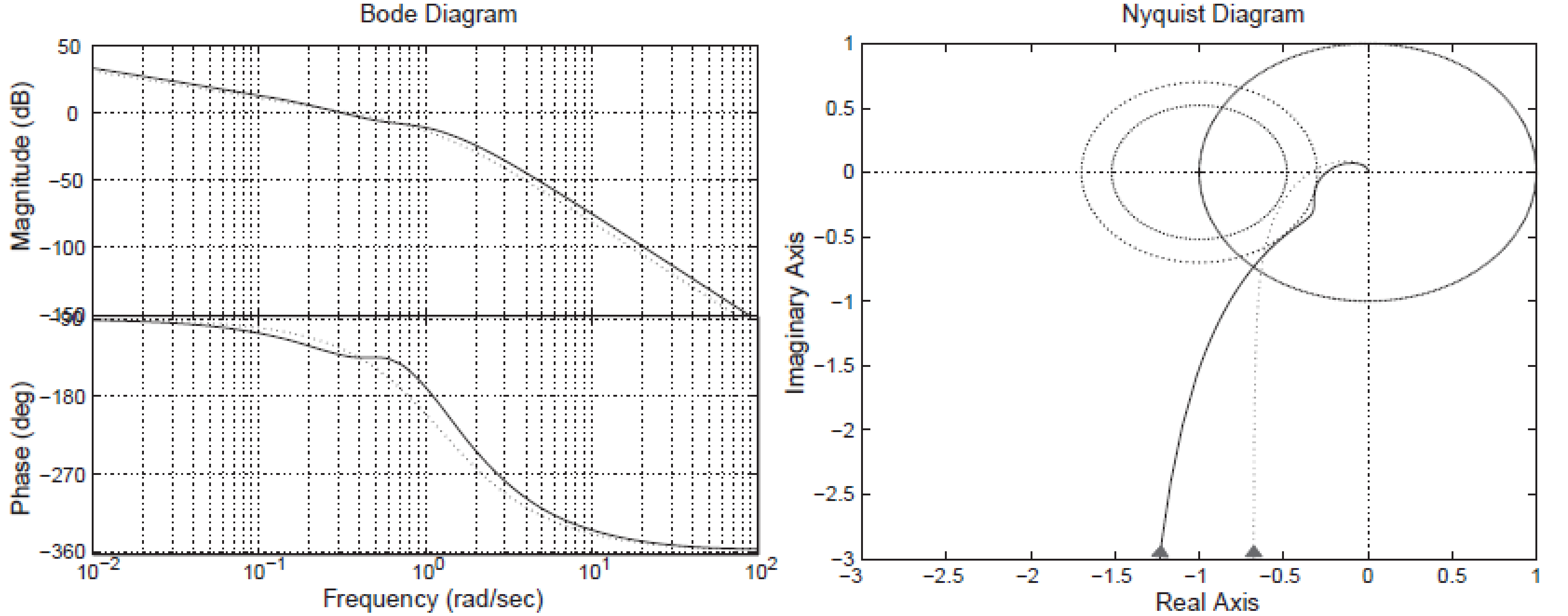
where n represents the current iteration number. With the new θ_n, after the relay test, the corresponding frequency ω_n can be recorded.

(4) Compare ω_n with ω_c. If $|\omega_n - \omega_c| < \delta$, quit iteration. Otherwise, go to Step 3. Here, δ is a small positive number.

A high order plant

$$P(s) = \frac{1}{(s+1)^5}$$

The PID controllers designed by different tuning methods	
Modified Ziegler-Nichols	The proposed method
$C(s) = 1.131(1 + \frac{1}{3.124s} + 0.781s)$	$C(s) = 0.921(1 + \frac{1}{1.961s} + 1.969s)$



(a) Comparison of Bode plots   (b) Comparison of Nyquist plots

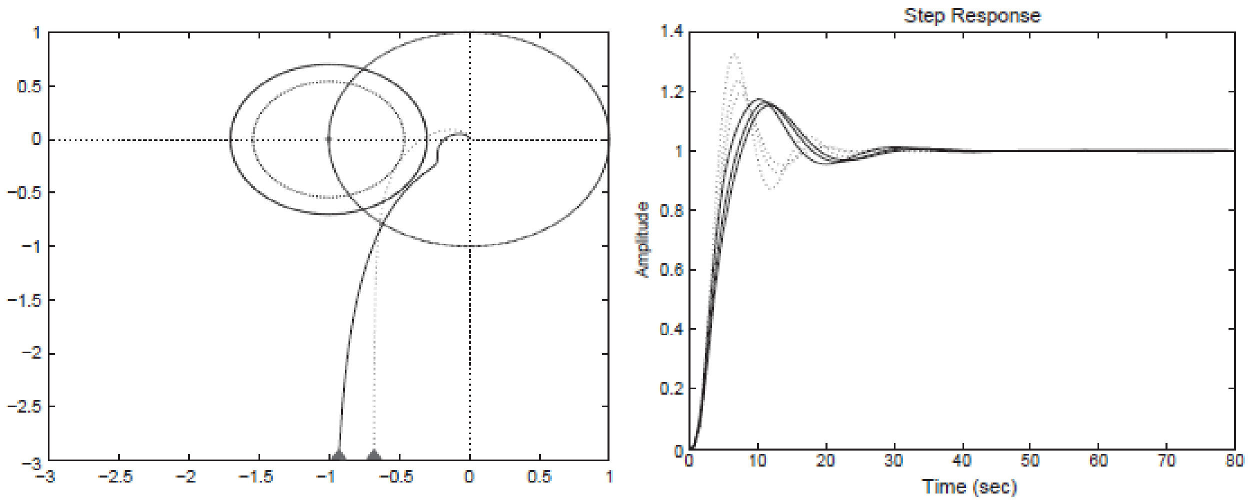
Figure: Frequency responses of the closed-loop system controlled by two PID controllers. (Dashed line: The modified Ziegler-Nichols, Solid line: The proposed PID controller)

A third order plant with an integrator

$$P(s) = \frac{1}{s(s+1)^3}$$

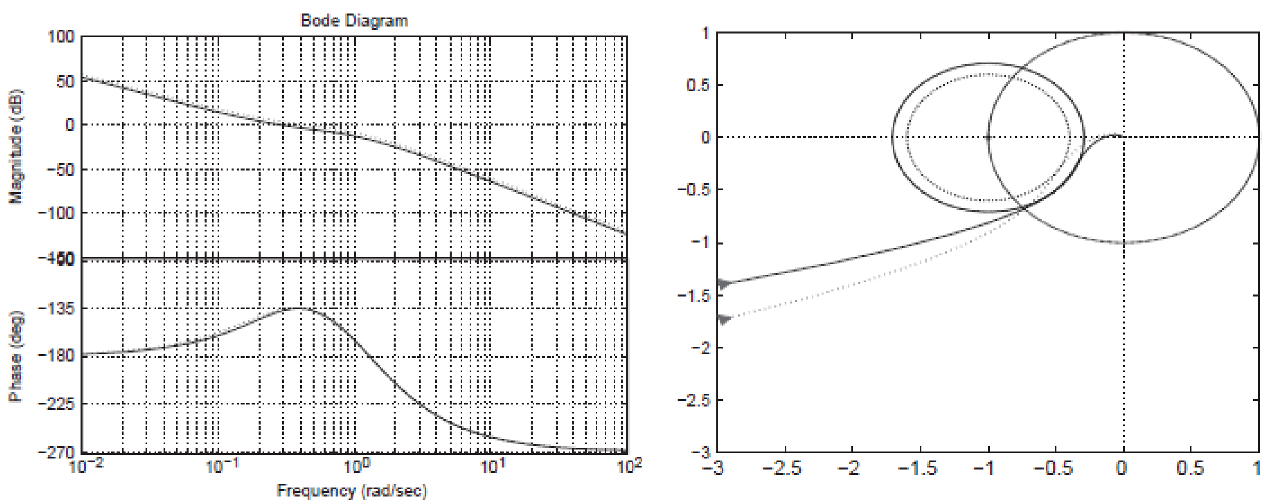
A third order plant with an integrator

$$C(s) = 0.33\left(1 + \frac{1}{6.53s} + 1.89s\right)$$



(a) Comparison of Nyquist plots
(b) Comparison of step responses

Figure: Comparisons of closed-loop frequency responses and step responses (Dashed line: The modified Ziegler-Nichols, Solid line: The proposed PID controller. For both schemes, gain variations 1, 1.1, 1.3 are considered in step responses



(a) Comparison of Bode plots
(b) Comparison of Nyquist plots

Figure: Frequency responses of the closed-loop system controlled by two PID controllers. (Dashed line: The modified Ziegler-Nichols, Solid line: The proposed PID controller)

Problem setup

- Integer order plants
- The plant gain and phase at the desired tangent frequency are identified by several relay feedback tests
- Iso damping property

Auto-tuning formulae for FO PI controllers

$$C(s) = K_p\left(1 + \frac{K_i}{s^\lambda}\right)$$

$$\begin{cases} K_p = \dfrac{\cos(\phi_m)}{|p(j\omega_c)|\sqrt{1 + 2K_i\omega_c^{-\lambda}\cos(\lambda\pi/2) + (K_i\omega_c^{-\lambda})^2}}. \\[2ex] K_i = \dfrac{-\tan(\phi)}{\omega_c^{-\lambda}(\sin(\lambda\pi/2) + \cos(\lambda\pi/2)\tan(\phi))}, \qquad \phi = \phi_m - \pi - \angle P(j\omega_c). \\[2ex] K_i = \dfrac{-(\lambda\sin(\frac{\lambda\pi}{2}) + 2\cos(\frac{\lambda\pi}{2})s_p(\omega_c)) + \sqrt{\triangle}}{2\omega_c^{-\lambda}s_p(\omega_c)}, \end{cases}$$

Problem setup

- Integer order plants
- The plant gain and phase at the desired tangent frequency are identified by several relay feedback tests
- Iso damping property

Auto-tuning formulae for FO [PI] controllers

$$C(s) = K_p\left(1 + \frac{K_i}{s}\right)^{\lambda}$$

$$\begin{cases} K_p = \dfrac{\cos(\phi_m)}{|P(j\omega_c)|\left(\sqrt{1 + K_i^2/\omega_c^2}\right)^{\lambda}}. \\[2ex] K_i = \omega_c \tan(\varphi), \qquad \phi = \phi_m - \pi - \angle P(j\omega_c). \\[2ex] K_i = \dfrac{-\lambda\omega_c \pm \omega_c\sqrt{\lambda^2 - 4s_p^2(\omega_c)}}{2s_p(\omega_c)}, \end{cases}$$

A high order plant

$$P(s) = \frac{1}{(s+1)^5}$$

The FO PI controllers	The FO [PI] controllers
$C(s) = 0.5616(1 + \frac{0.1869}{s^{1.3}})$	$C(s) = 0.4464(1 + \frac{0.1815}{s})^{1.86}$

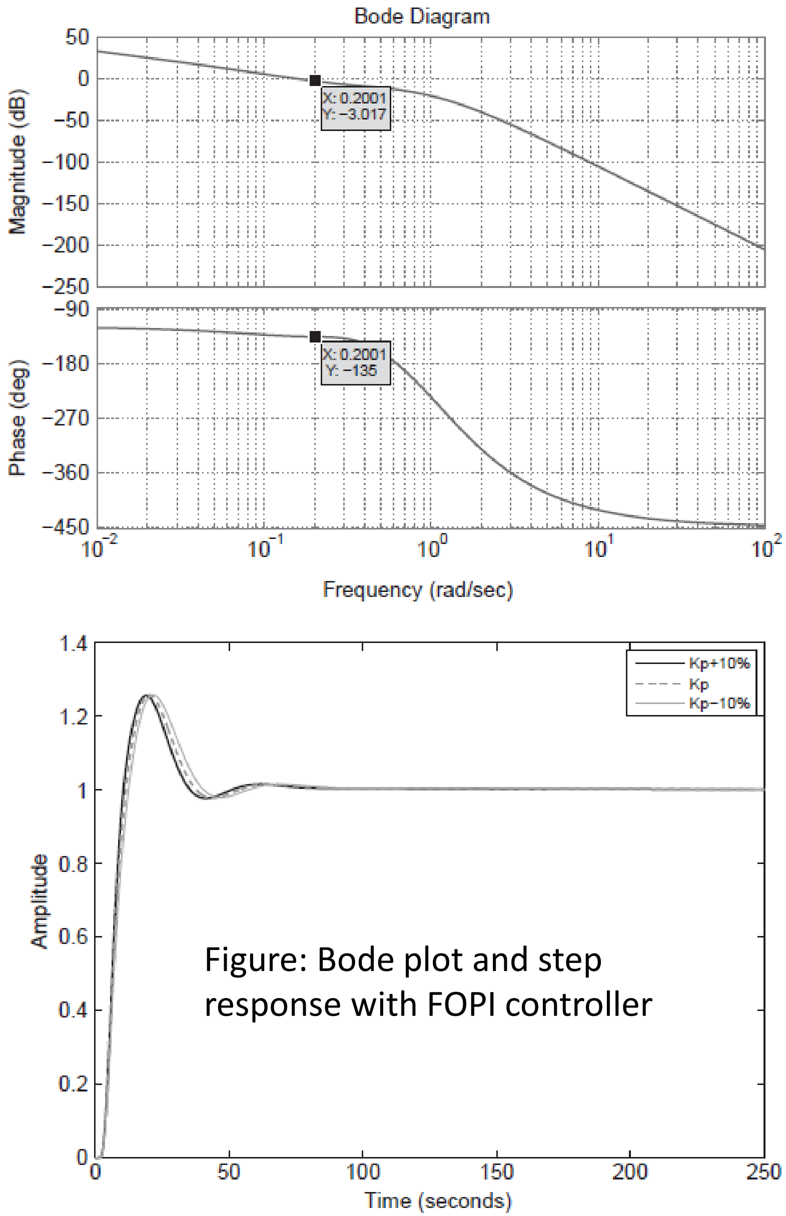


Figure: Bode plot and step response with FOPI controller

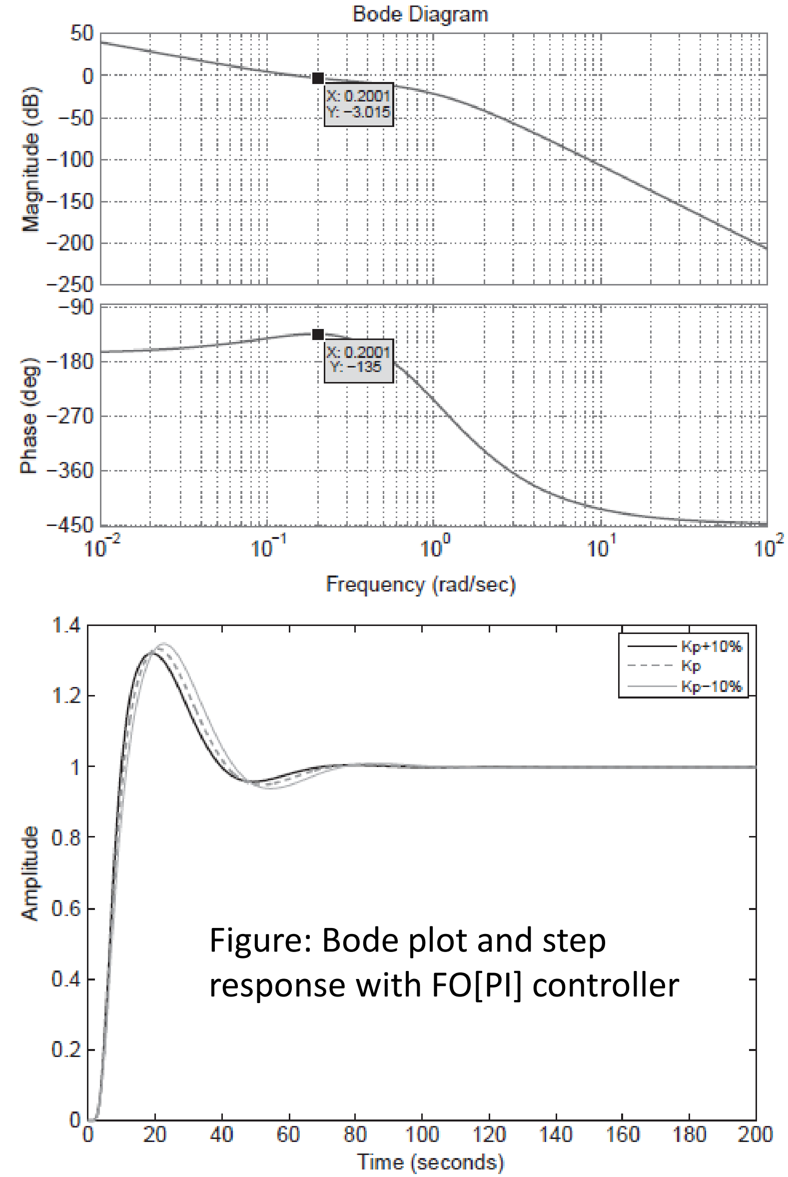


Figure: Bode plot and step response with FO[PI] controller

More examples are available in the book

Plant with an integrator

$$P(s) = \frac{1}{s(s+1)^3}$$

Plant with time delay

$$P(s) = \frac{1}{(s+1)^3} e^{-s}$$

This is the end of session II

Questions?