Turkish National Meeting on Automatic Control
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A Tutorial on Fractional Order Motion Control

Part IV: Stability and Feasibility

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- Fractional Order Motion Controls

John Wiley & Sons, Inc.

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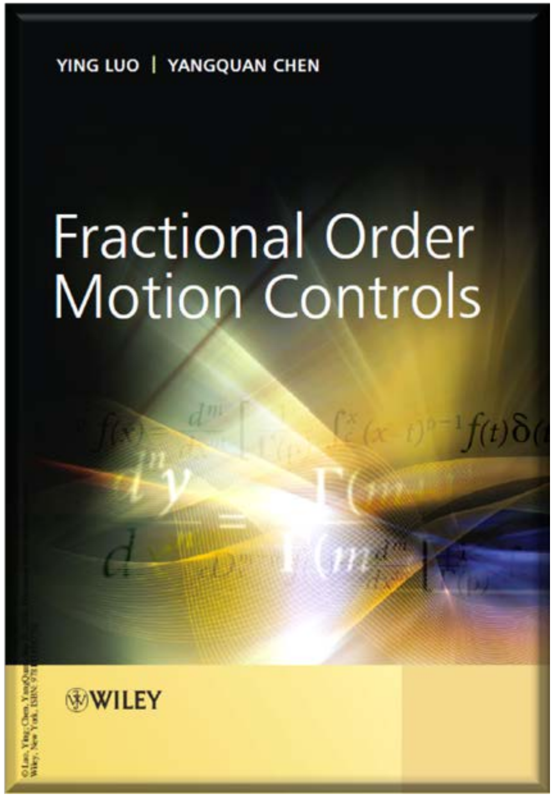

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Scope of the book

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Plant

$$P(s) = \frac{K}{Ts+1}e^{-Ls}$$

Controller

$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

The gain-phase margin tester

$$M_T(A,\phi) = Ae^{-j\phi}$$

A is the boundary of gain margin, is the boundary of phase margin

Definition: Gain-phase margin tester

M_T in figure below is a gain-phase master which provides information for plotting the boundaries of constant gain margin and phase margin in the parameter plane.

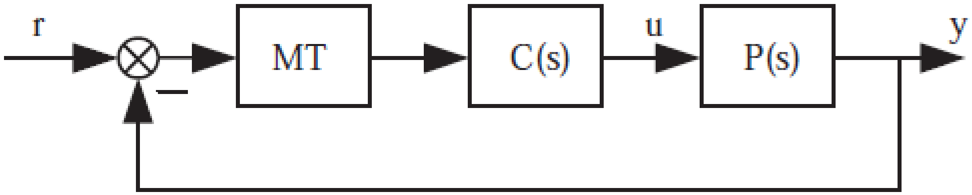


Figure: The feedback control system with the gain-phase margin tester

Open loop

$$G(s) = M_T(A,\phi)C(s)P(s)$$

Closed loop

$$\Phi(s) = \frac{M_T(A,\phi)C(s)P(s)}{1 + M_T(A,\phi)C(s)P(s)}$$
$$= \frac{Ae^{-j\phi}Ke^{-Ls}(K_p s + K_i + K_d s\ )}{s(Ts+1) + Ae^{-j\phi}Ke^{-Ls}(K_p s + K_i + K_d s\ )}$$

Property

Assuming $\phi = 0$ for M_T, the controller parameters can be obtained satisfying a given gain margin A. Vise versa, assuming $\mathrm{A} = 1$ for M_T, one can obtain the controller parameters for a given phase margin ϕ.

Definition: IRB (Infinity root boundary):

$$D\left(K_p, K_i, K_d, A, \phi; s = \infty\right) = 0$$

$$\Rightarrow K_d = \pm \frac{T}{AK}$$

Definition: RRB (Real root boundary):

$$D\left(K_p, K_i, K_d, A, \phi; s = 0\right) = 0$$

$$\Rightarrow K_i = 0$$

Definition: CRB (Complex root boundary):

$$D\left(K_p, K_i, K_d, A, \phi; s = j\omega\right) = 0$$

$$\Rightarrow \begin{cases} K_d = \dfrac{KAsin(\phi + \omega L)K_i - KA\omega K_p \cos(\phi + \omega L) - \omega}{KA\omega^2 \sin(\phi + \omega L)} \\ K_p = \dfrac{T\omega \sin(\phi + \omega L) - \cos(\phi + \omega L)}{KA} \\ K_i = \dfrac{\omega \sin(\phi + \omega L) - T\omega^2 \cos(\phi + \omega L)}{KA} + \omega^2 K_d \end{cases}$$

Property

The controller parameter boundaries of the stability region Q can be determined by the infinity root boundary (IRB), the real root boundary (RRB), and the complex root boundary (CRB)

$$k_d = \begin{cases} 0 & \text{for } (\alpha_n = \beta_n) \\ & \quad \text{or } (\alpha_n > \beta_n \text{ and } \mu > \alpha_n - \beta_n) \\ \pm a_n / b_n & \text{for } (\alpha_n > \beta_n \text{ and } \mu = \alpha_n - \beta_n) \\ \text{none} & \text{for } (\alpha_n > \beta_n \text{ and } \mu < \alpha_n - \beta_n). \end{cases} \qquad (8)$$

An Algorithm for Stabilization of Fractional-Order Time Delay Systems Using Fractional-Order PID Controllers

Serdar Ethem Hamamci

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 52, NO. 10, OCTOBER 2007

Definition: the relative stability line and surface

Given one specification – phase margin $\varphi = \varphi_m$ $(A = 1)$, and from CRB, a relative stability line can be drawn in the (K_i, K_d) parameter-plane as $\omega \rightarrow \omega_0$ from zero with a certain fixed $K_{d1} \in [-T/K, T/K]$. ω_0 is the maximum frequency guaranteeing the pre-specified phase margin with the fixed Kd1 on the relative stability line. Sweeping all the K_d in $[-T/K, T/K]$, a surface in the 3D parameter-space can be generated satisfying the pre-specified phase margin φ_m, which is called the relative stability surface. Since there exists a maximum frequency ω_0 for every relative stability line, the frequency boundary of the relative stability surface can be found.

Property

The frequency ω of every point on the relative stability line can be used as the gain crossover frequency with the corresponding PID controller parameters

Definition: the flat phase stable point

All the points with PID parameters on the relative stability curve can be tested by the equation $\frac{d\phi}{d\omega} = 0$. If a certain point with the PID parameters (K_p, K_i, K_d) can be found to guarantee the relationship, this point is called the flat phase stable point.

Property

With this flat phase constraint, the open-loop system phase can maintain almost the same value when the loop gain changes in a certain interval, namely, the system with this designed PID is robust to the loop gain variations. The overshoots of the step responses are almost the same with the variations of loop gain in certain range. Hence, the control performance of the system with the designed PID controller degrades gracefully when the steady-state gains of the plant and the PID controller change.

Example

$K_d = 0.5$
Complete stability region

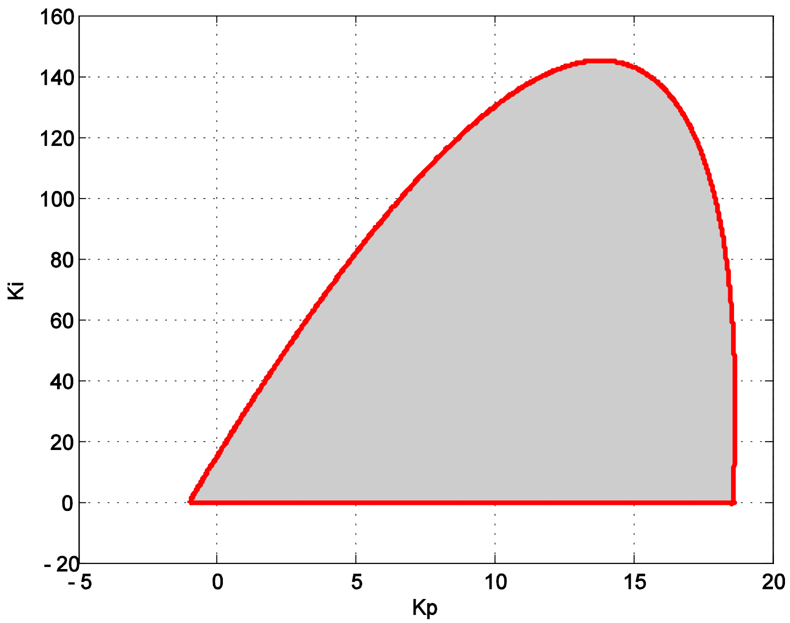
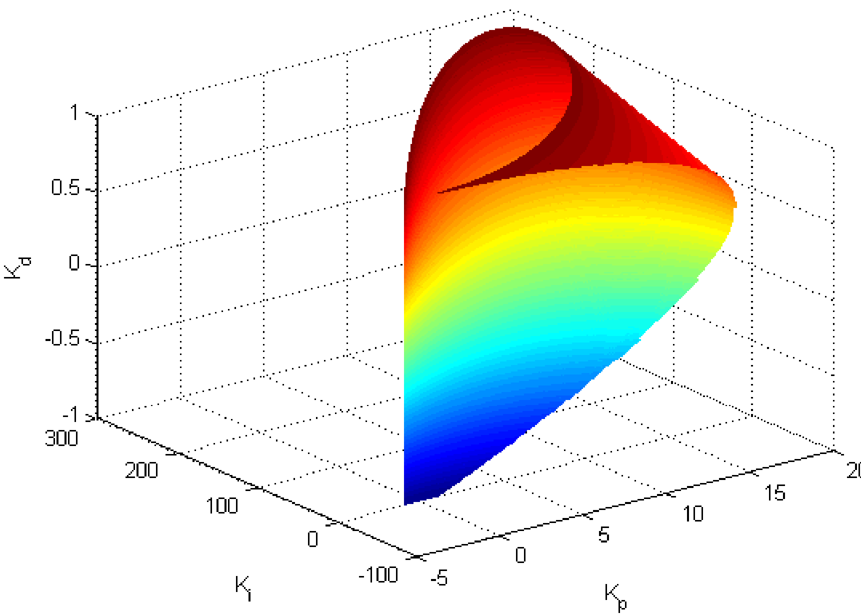


Figure: Stability region of *Ki* with respect to *Kp* with *Kd* = 0.5



Complete stability region of *Ki*, *Kp* and *Kd*

Example

$K_d = 0.5$
$\varphi_m = 50°$

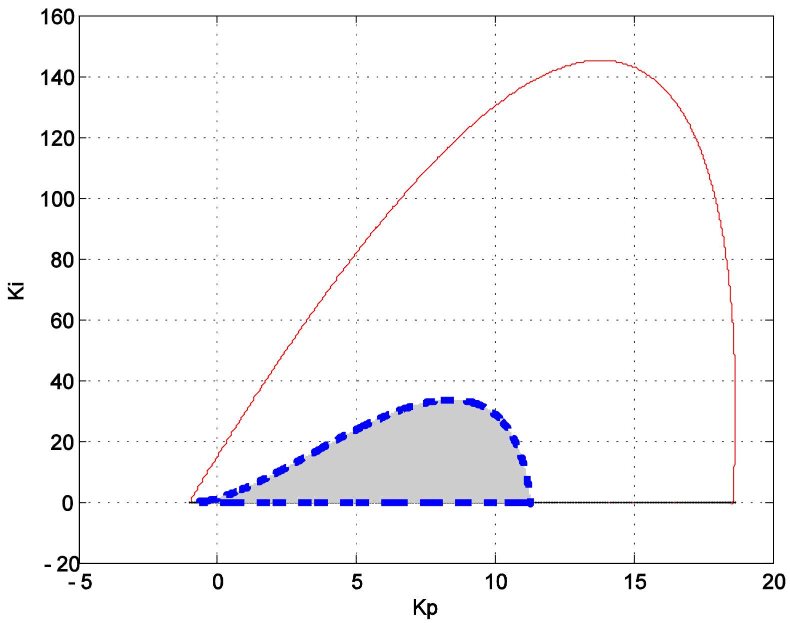


Figure: Stability region of *Ki* with respect to *Kp* with *Kd* = 0.5 and $\varphi_m = 50°$

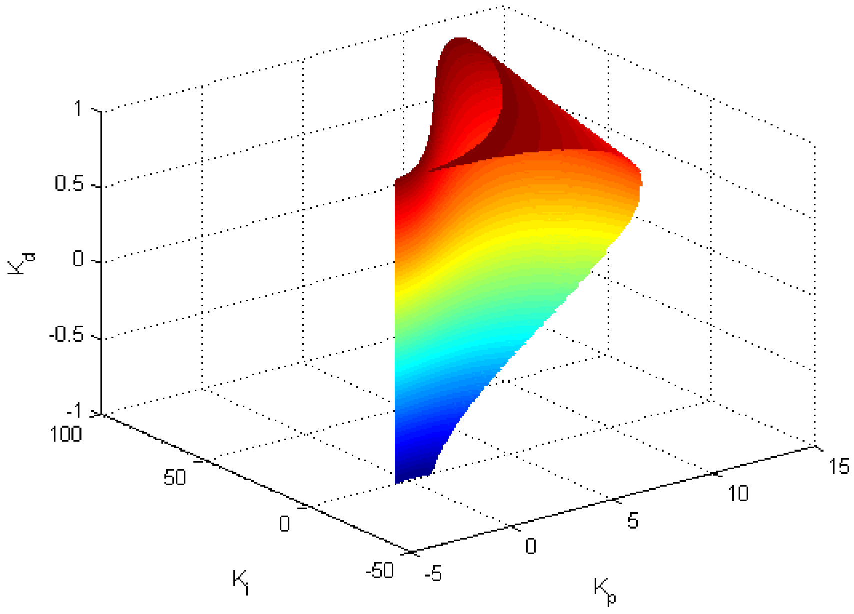
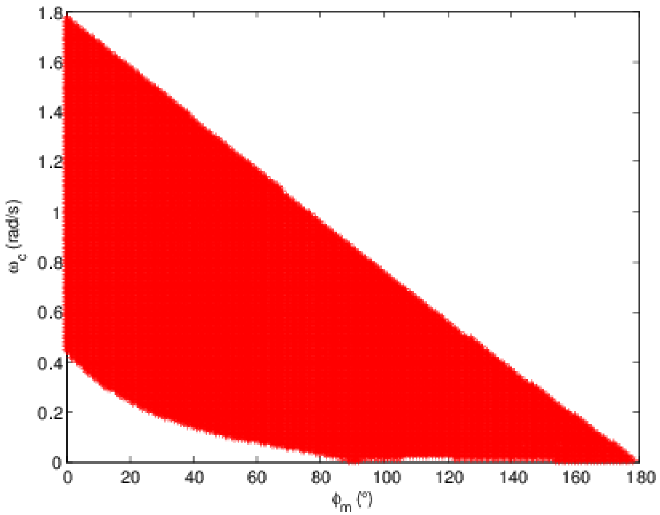
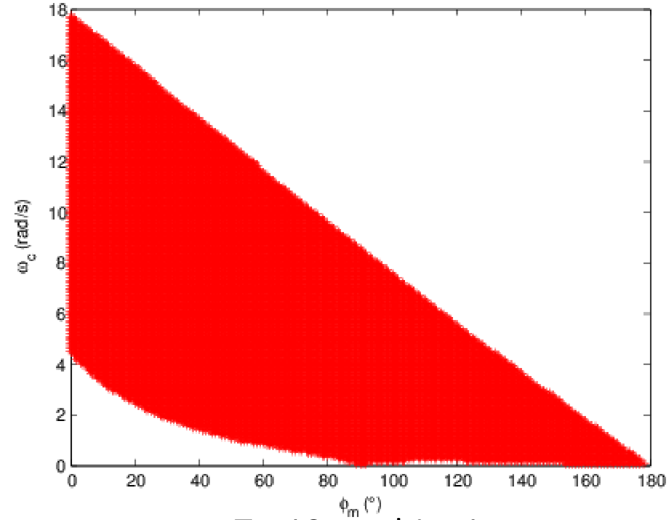


Figure: Complete stability region of *Ki*, *Kp* and *Kd* and $\varphi_m = 50°$

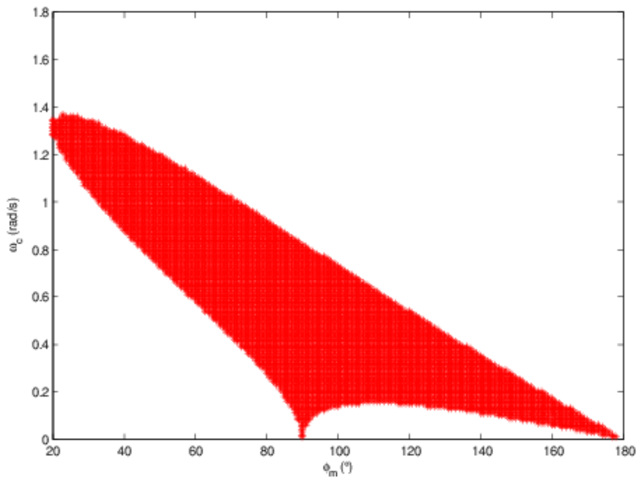
Example: Achievable region for PID



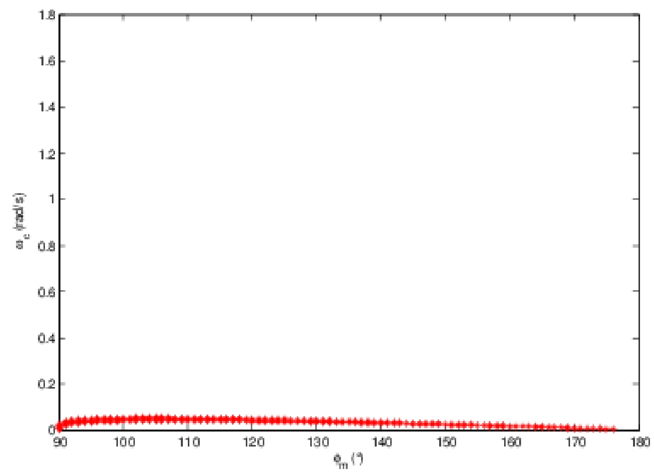
T = 1s and L = 0.1s



T = 10s and L = 1s



T = 1s and L = 1s



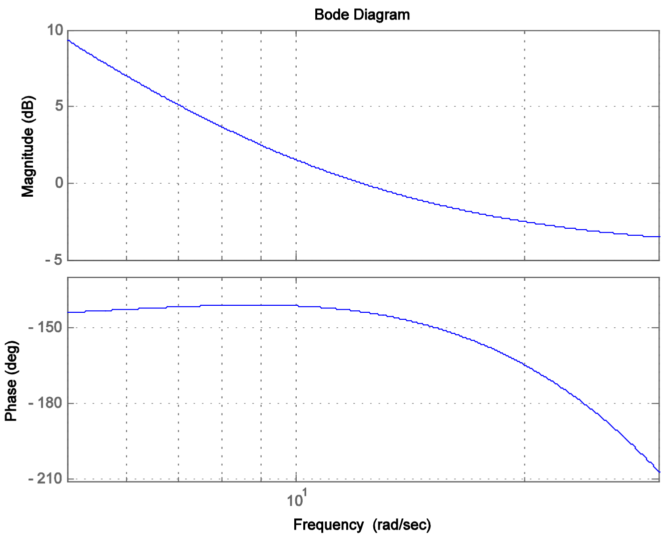
T = 1s and L = 10s

The plant

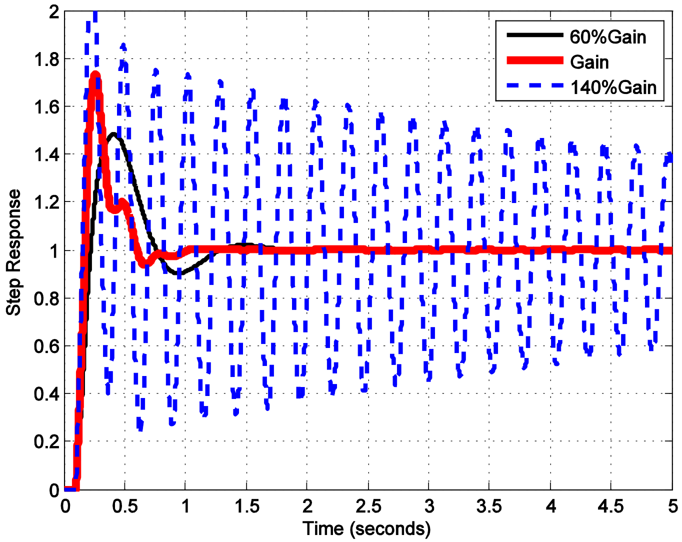
$$P(s) = \frac{1}{s+1} e^{-0.1s}$$

The PID controllers designed by different tuning methods

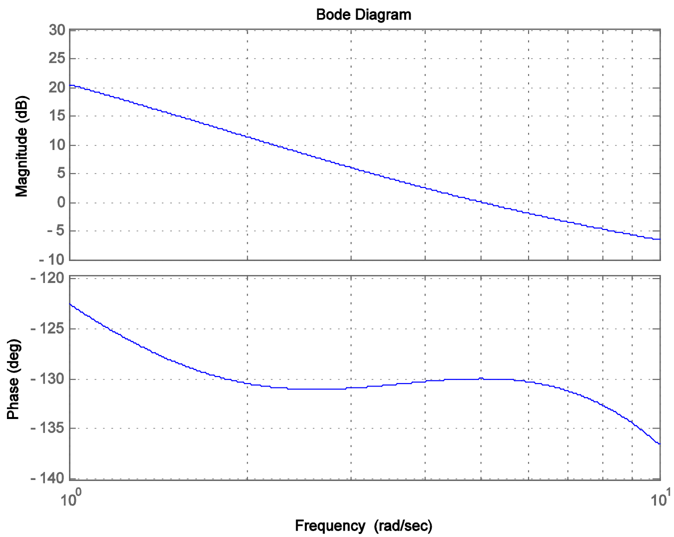
Ziegler-Nichols	The flat phase constraint
$K_p = \frac{1.2T}{KL}, K_i = \frac{K_p}{2L}, K_d \frac{K_p L}{L}$ $C(s) = 12 + \frac{60}{s} + 0.6s$	$C(s) = 4.71 + \frac{14.48}{s} + 0.19s$



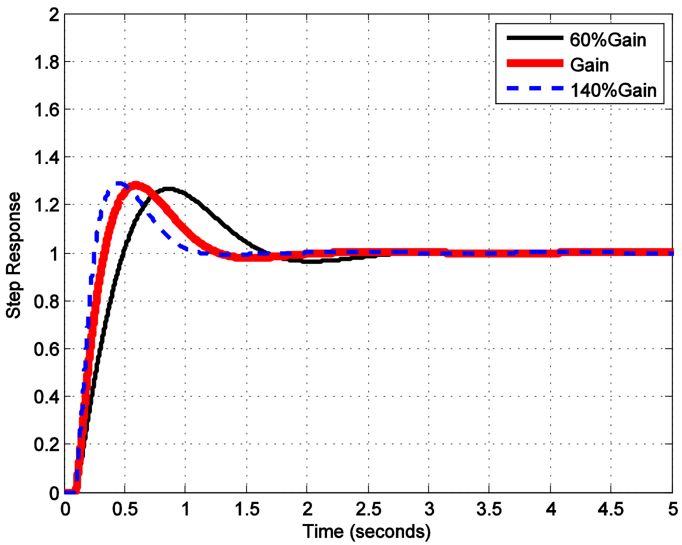
Bode plot of ZN PID



Step response of ZN PID



Bode plot of flat phase PID



Step response of flat phase PID

The controller

$$C(s) = K_p + \frac{K_i}{s^r}$$

the characteristic equation of the closed-loop system

$$D\left(K_p, K_i, A, \phi; s\right) = s^r(Ts + 1) + Ae^{-j\phi}Ke^{-Ls}(K_p s^r + K_i)$$

RRB

$$D\big(K_p, K_i, K_d, A, \phi; s = 0\big) = 0$$

$$\Rightarrow K_i = 0$$

CRB

$$D\big(K_p, K_i, K_d, A, \phi; s = j\omega\big) = 0$$

$$\Rightarrow \begin{aligned} & D(K_p, K_i, r, A, \phi; j\omega) = (j\omega)^r (jT\omega + 1) + Ae^{-j\phi} e^{-j\omega L} K(K_p (j\omega)^r + K_i) \\ & = \omega^r \cos\frac{r\pi}{2} - T\omega^{1+r} \sin\frac{r\pi}{2} \\ & \quad + AK \cos(\phi + \omega L)(K_i + K_p \omega^r \cos r\pi 2) + AK \sin(\phi + \omega L) K_p \omega^r \sin\frac{r\pi}{2} \\ & \quad + j(T\omega^{1+r} \cos\frac{r\pi}{2} + \omega^r \sin\frac{r\pi}{2} + AK \cos(\phi + \omega L) K_p \omega^r \sin\frac{r\pi}{2} \\ & \quad - AK \sin(\phi + \omega L)(K_i + K_p \omega^r \cos r\pi 2)) \\ & = 0, \qquad (12 \end{aligned}$$

CRB – cont.

$\Rightarrow$

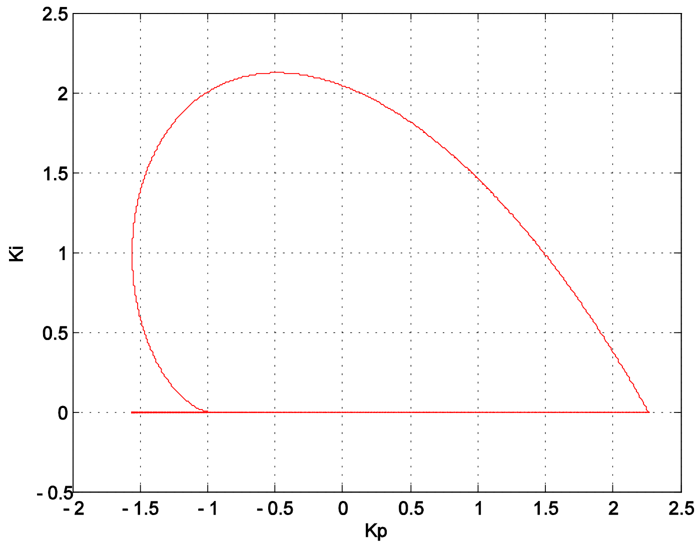
$$K_p = \frac{-(B_1 S_1 + B_2 C_1)}{AKS_2 \omega^r},$$

$$K_i = \frac{B - B_1 S_1 C_1 - B_2 C_1^2}{AKS_1} + \frac{B_1 S_1 C_2 + B_2 C_1 C_2}{AKS_2}.$$

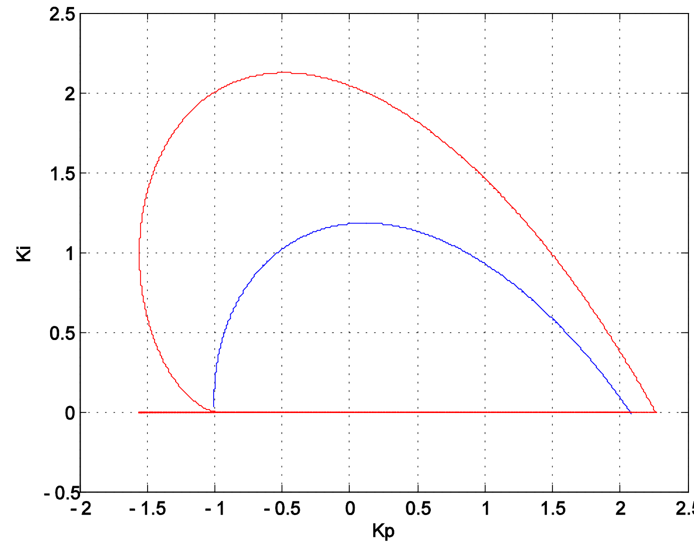
With plat phase constraint

$$\frac{\mathrm{d}\phi}{\mathrm{d}\omega} = \frac{(B_1^2 + B_2^2)(EF' - E'F) + (B_1' B_2 - B_1 B_2')(E^2 + F^2)}{(B_1 E + B_2 F)^2 + (B_1 F - B_2 E)^2} - L$$
$$= 0,$$

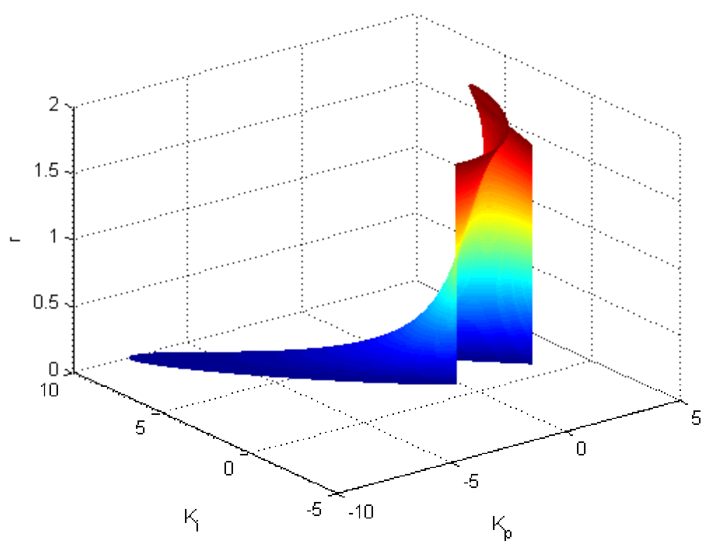
Stability region for FO PI



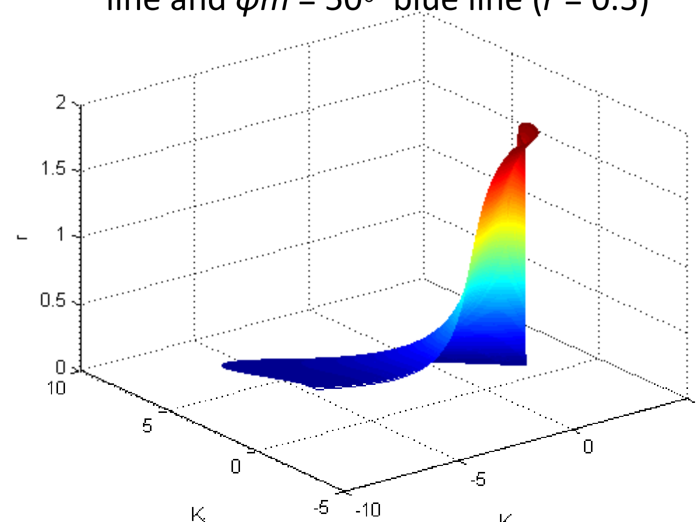
Stability boundary of *Ki* vs. *Kp* with *r* = 0.5



Stability region comparison of *Ki* vs. *Kp* with *φm* = 0 ◦ dark line and *φm* = 50◦ blue line (*r* = 0.5)

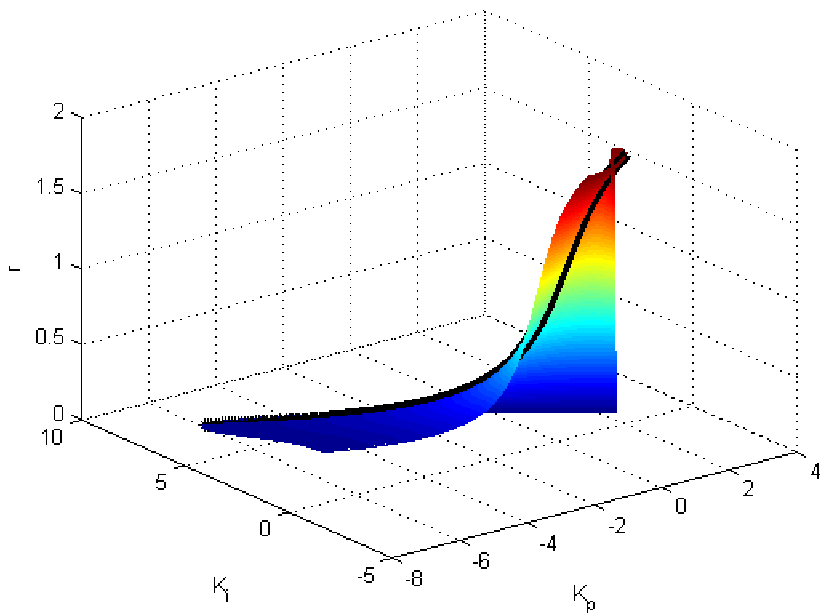


Complete stability region of Ki, Kp and r

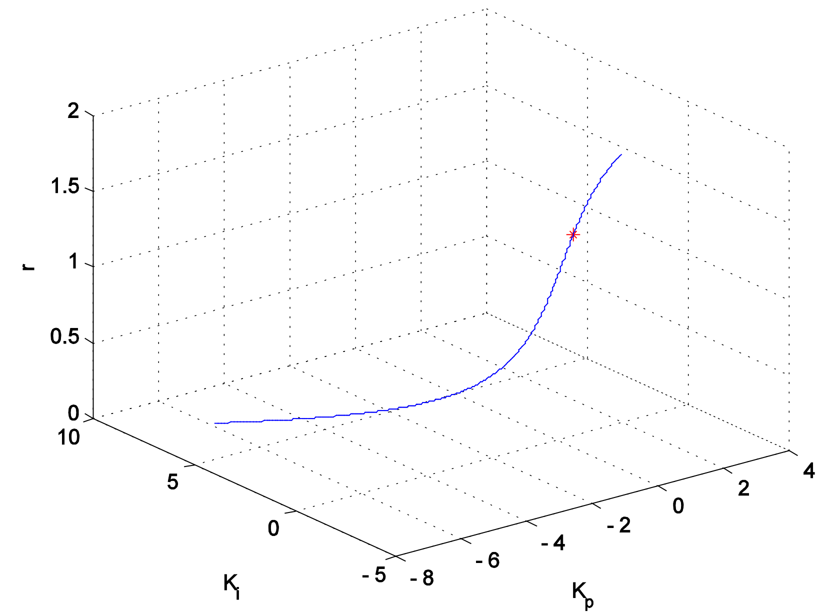


Three dimensional relative stability surface of Ki, Kp and r with ϕm = 50◦

Stability region for FO PI



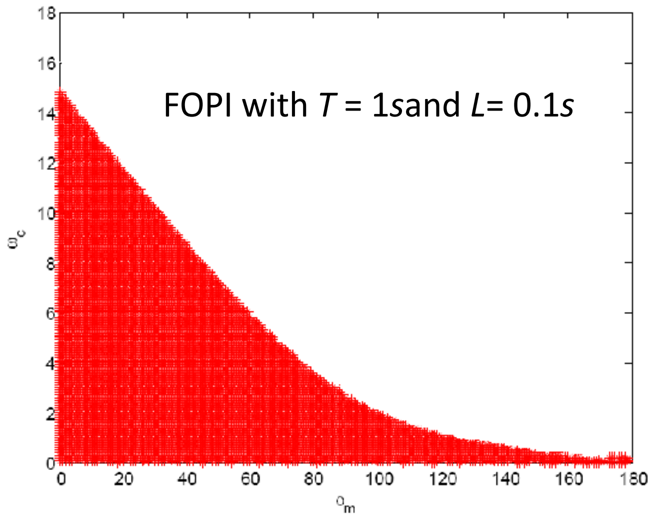
The relative stability curve on the relative stability surface

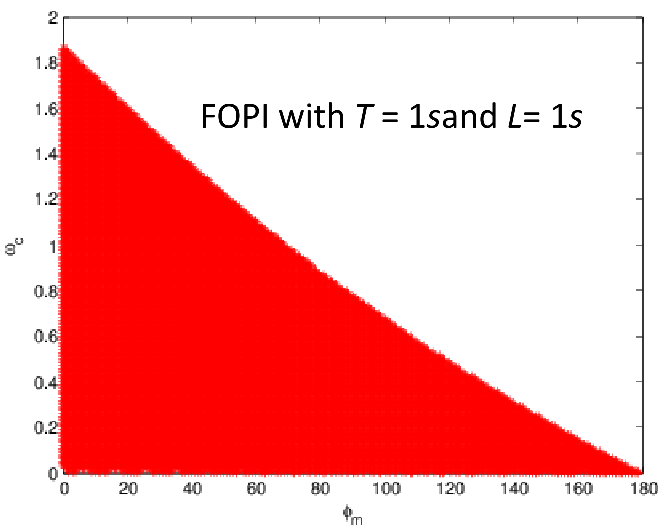


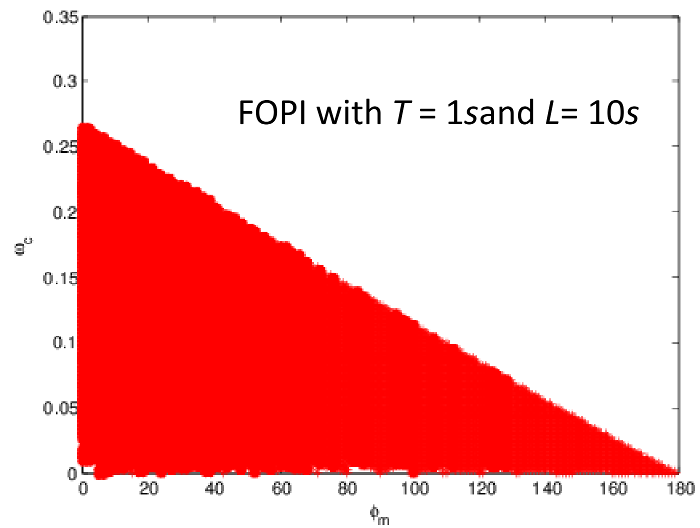
The flat phase stable point on the relative stability curve

Figure: The relative stability curve and the flat phase stable point in the 3D parameter space

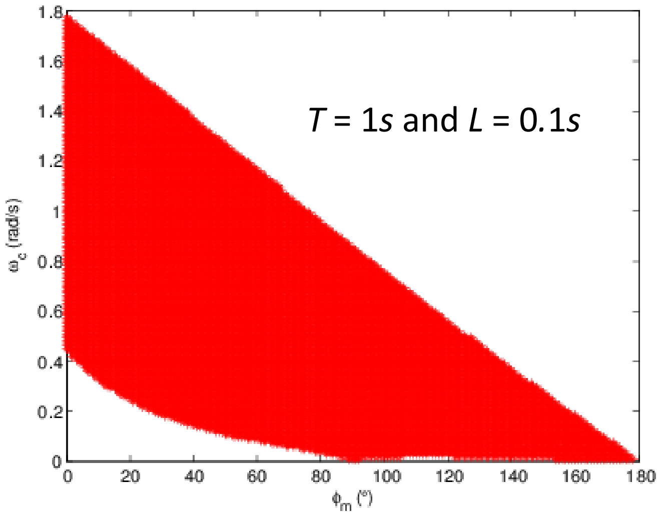
Example: Achievable region for FOPI

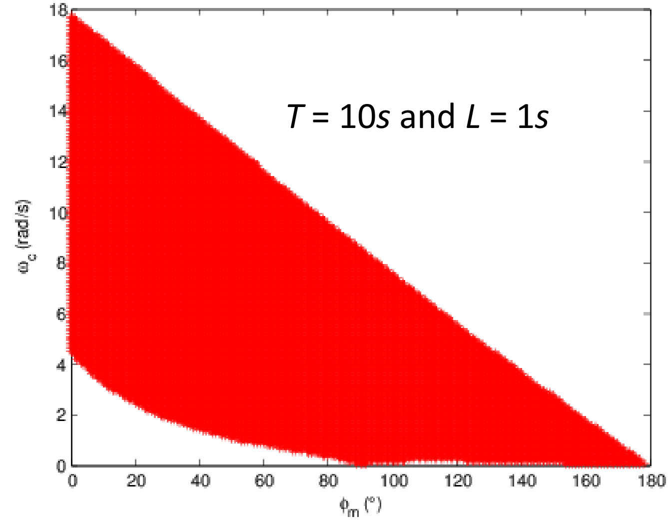


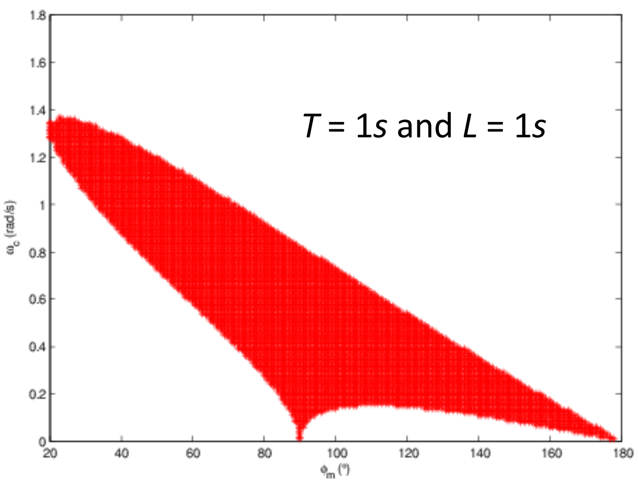


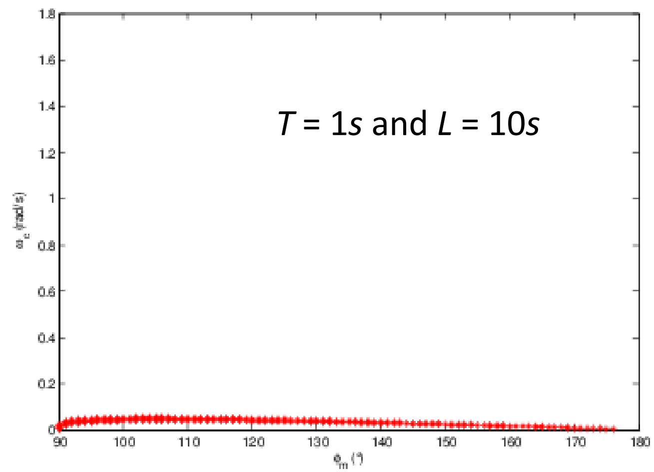


Recall: Achievable region for IOPID









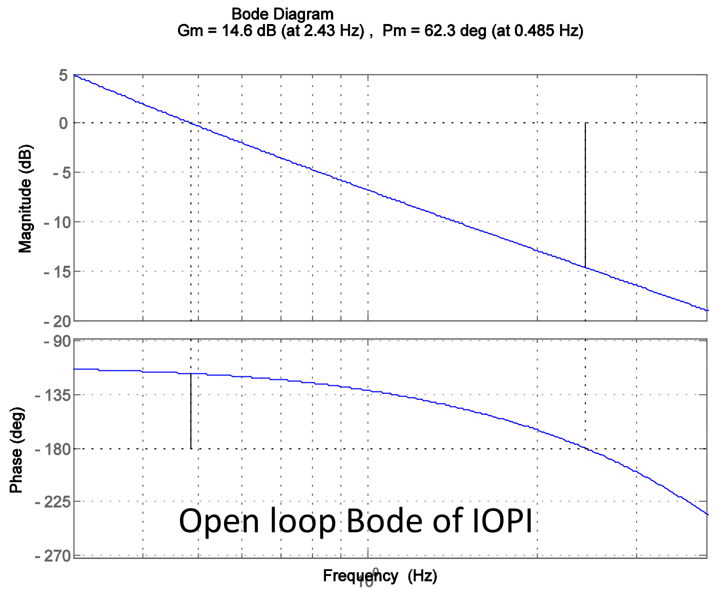
Conclusion

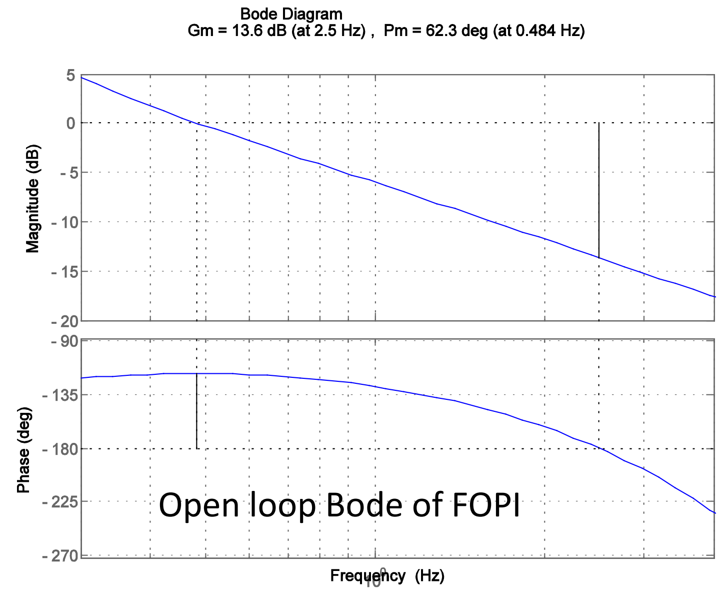
FO PI has larger achievable region than IOPID

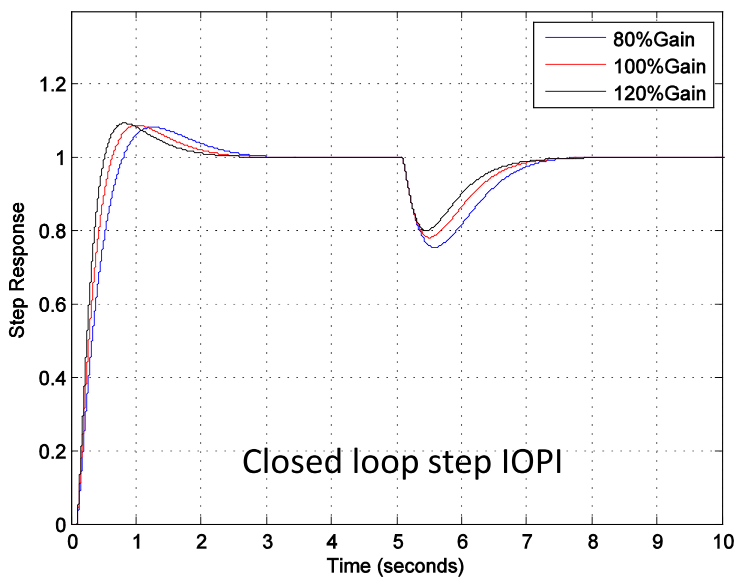
The plant

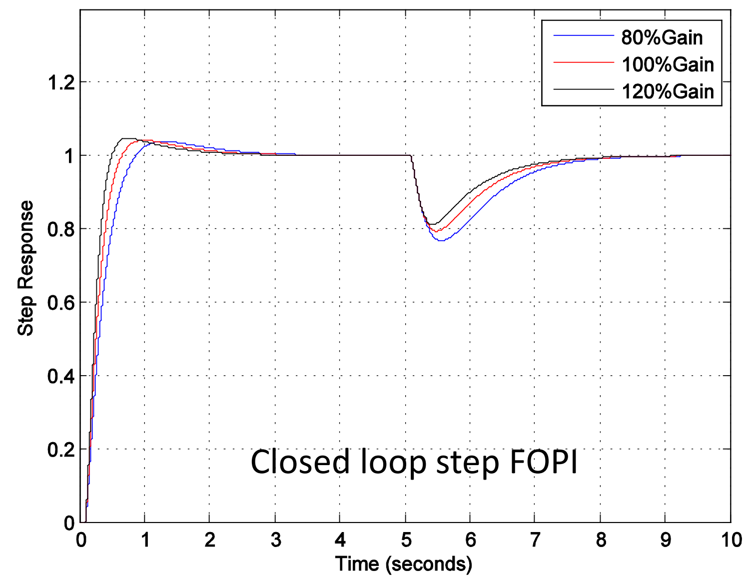
$$P(s) = \frac{1}{s+1}e^{-0.1s}$$

The PID controllers designed by different tuning methods	
AMIGO (Approximate M-constrained integral gain optimization)	The flat phase constraint
$K_p = \frac{1.5}{K} + [0.35 - \frac{LT}{(L+T)^2}]\frac{T}{KL}$ $K_i = \frac{K_p}{2L}(0.35L + \frac{13LT^2}{T^2 + 12LT + L^2})$ $C(s) = 2.82 + \frac{4.65}{s}$	$C(s) = 3.34 + \frac{6.20}{s^{1.44}}$


Bode Diagram
Gm = 14.6 dB (at 2.43 Hz) , Pm = 62.3 deg (at 0.485 Hz)
Magnitude (dB)
Phase (deg)
Frequency (Hz)
Open loop Bode of IOPI


Bode Diagram
Gm = 13.6 dB (at 2.5 Hz) , Pm = 62.3 deg (at 0.484 Hz)
Magnitude (dB)
Phase (deg)
Frequency (Hz)
Open loop Bode of FOPI


80%Gain
100%Gain
120%Gain
Step Response
Time (seconds)
Closed loop step IOPI


80%Gain
100%Gain
120%Gain
Step Response
Time (seconds)
Closed loop step FOPI

This is the end of session IV

Questions?