Turkish National Meeting on Automatic Control
(TOK 2013) , Sept. 25, 2013, Malatya, Turkey

A Tutorial on Fractional Order Motion Control

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Part I: Overview of the Book and Fundamentals of Fractional Order Controls

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- Fractional Order Motion Controls

John Wiley & Sons, Inc.

Hardcover, 454 pages, December 2012

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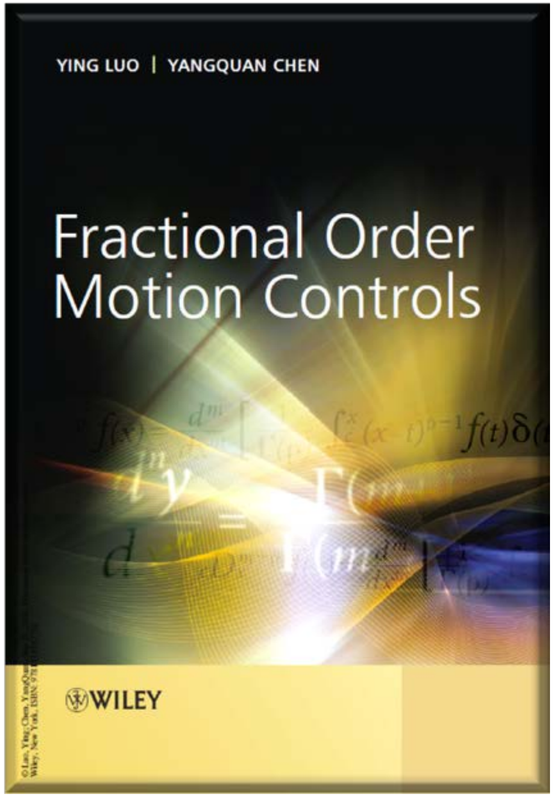
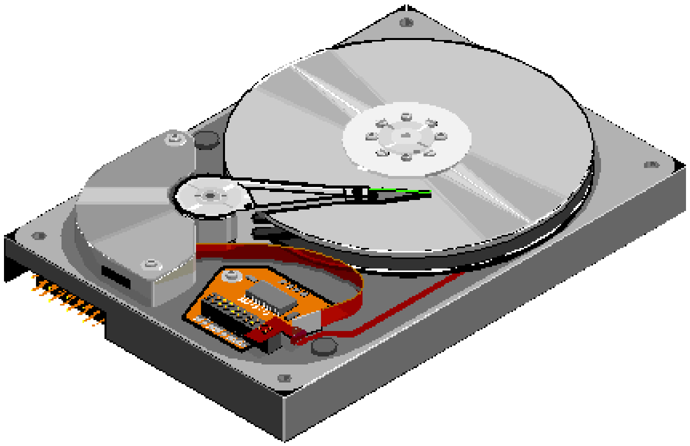


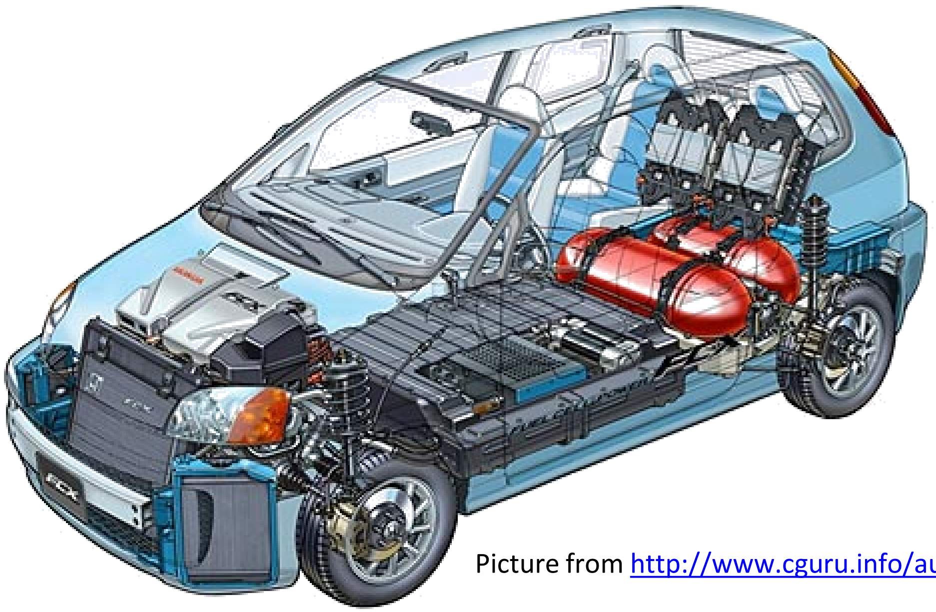
Scope of the book

Contributions

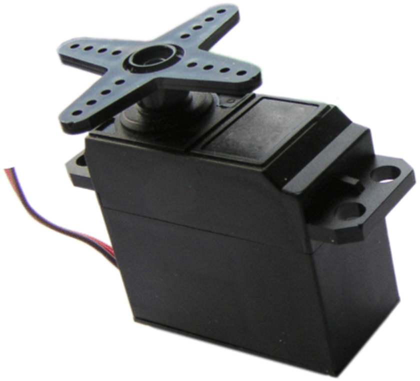
- In the velocity and position motion systems, fractional calculus is applied to the FO modeling and FO controllers design in systematic ways to achieve better performances than the designed integer order controllers using traditional optimization methods. Stability and feasibility issues are analyzed and discussed for the fractional order controllers design.
- FO disturbance observer, FO adaptive feed-forward scheme, FO adaptive control and FO periodic adaptive learning control for external disturbance compensation are presented.
- This book documents the very first optimization approach of the proposed FO conditional integrator for nonlinear system controls. Meanwhile, some efforts of the fractional order control on nonlinearities, for example, friction and backlash, are presented.
- Simulation illustrations and/or experimental validations are demonstrated for all the proposed control schemes and approaches. The real applications for FO motion controls on unmanned aerial vehicles and hard-disk drives are presented in this book.

Applications





Picture from http://www.cguru.info/automobile.htm



Collection of resources for control tutorial

Control Tutorials for MATLAB and Simulink
http://ctms.engin.umich.edu/CTMS/index.php?aux=Home

Collection of resources for fractional calculus

Fractional Calculus Day @ UCMerced
http://mechatronics.ucmerced.edu/node/68

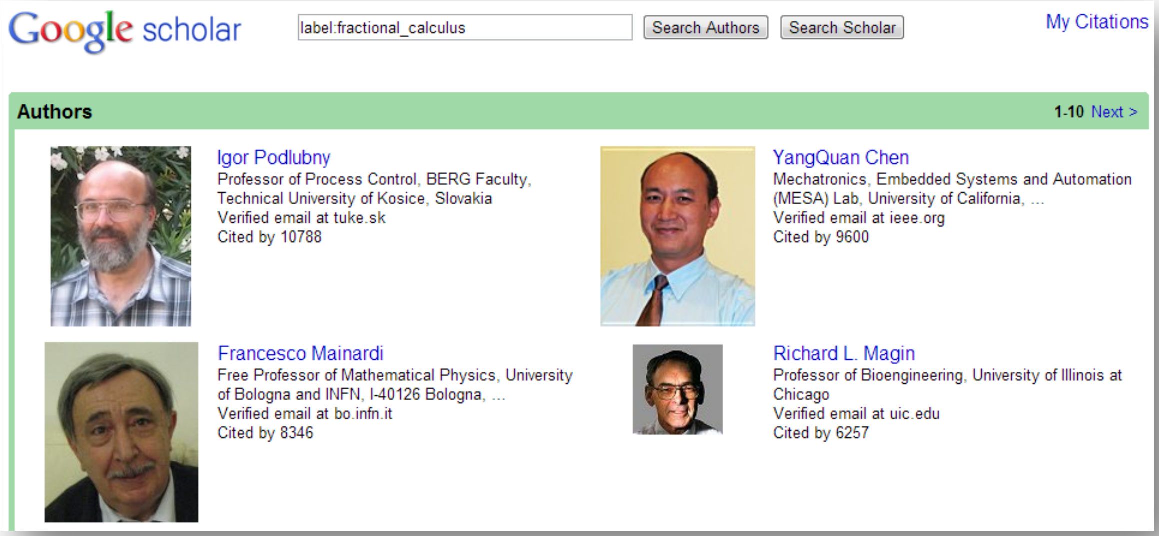
Applied Fractional Calculus (AFC) @ UCMerced
http://mechatronics.ucmerced.edu/research/applied-fractional-calculus

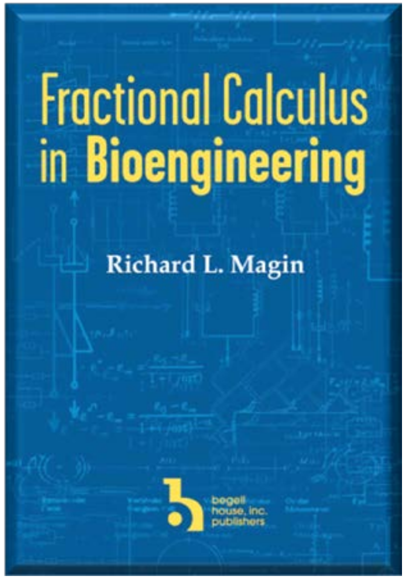
Applied Fractional Calculus (AFC) @ USU
http://mechatronics.ece.usu.edu/foc/

Collection of resources for fractional calculus

Richard Magin, Fractional Calculus in Bioengineering, ISBN: 978-1-56700-215-7, 684 pages, 2006.

Igor Podlubny, Fractional Differential Equations, 1st Edition, An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications, 1998





- Crises in the foundations of mathematics push the development of math
- Paradox
  - What does n=1/2 mean in the derivative of a function?

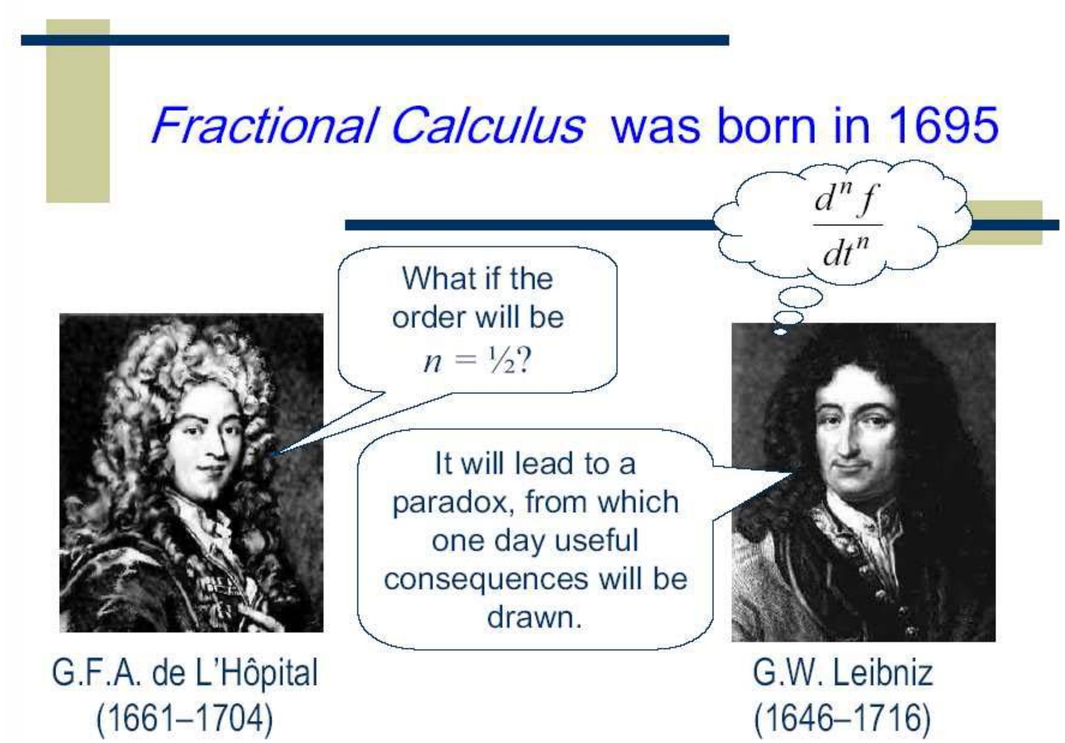


Figure: The born of fractional calculus

Definition 1

Fractional Order **Cauchy** integral formula

$$D^{\alpha} f(t) = \frac{\Gamma(\alpha+1)}{j2\pi} \int_C \frac{f(\tau)}{(\tau - t)^{\alpha+1}} d\tau, \tag{1}$$

where C is the closed-path that encircles the poles of the function f (t).

Definition 2

Grünwald-Letnikov definition

$${}_aD_t^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\alpha}{j} f(t - jh), \tag{2}$$

where $\binom{\alpha}{j}$ are the binomial coefficients; the subscripts to the left and right of D are the lower- and upper-bounds in the integral. The value α can be positive or negative, corresponding to differentiation and integration, respectively.

Definition 3

Riemann-Liouville definition

$$ {}_aD_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad (3) $$

where $0 < \alpha < 1$, and a is the initial value. Let a = 0, the notation of integral can be simplified to $D_t{}^{-\alpha} f(t)$.

Definition 4

Caputo definition

$$ {}_0D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f^{(m+1)}(\tau)}{(t-\tau)^{\alpha}} d\tau, \quad (4) $$

where $\alpha = m + \gamma$, m is an integer and $0 < \gamma \leq 1$.

Fractional order differential equations

$$\begin{aligned} & a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \cdots + a_0 D^{\alpha_0} y(t) \\ & \quad = b_m D^{\beta_m} u(t) + b_{m-1} D^{\beta_{m-1}} u(t) + \cdots + b_0 D^{\beta_0} u(t), \end{aligned} \tag{5}$$

Laplace transform of a fractional order derivative operator

$$D^\alpha \to s^\alpha \tag{6}$$

Where $-1 \leq \alpha \leq n$.

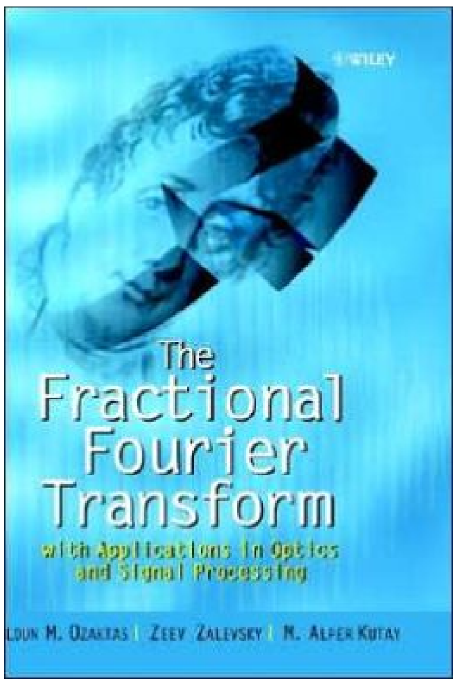
Laplace transform of the above differential equation

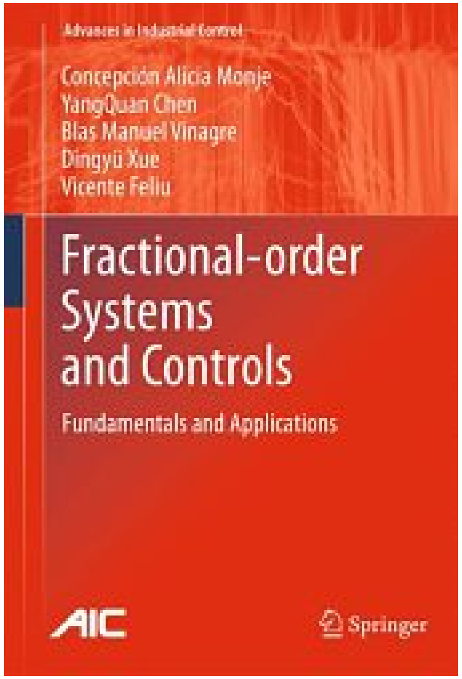
$$G(s) = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]} = \frac{b_1 s^{\gamma_1} + b_2 s^{\gamma_2} + \cdots + b_m s^{\gamma_m}}{a_1 s^{\eta_1} + a_2 s^{\eta_2} + \cdots + a_{n-1} s^{\eta_{n-1}} + a_n s^{\eta_n}}. \tag{7}$$

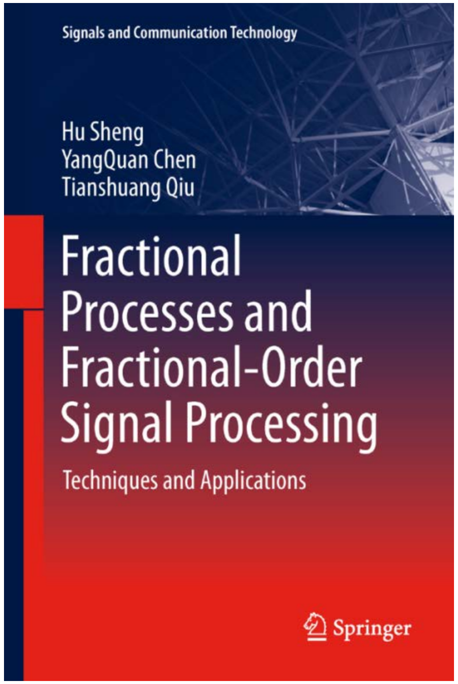
Fourier transform of a fractional order derivative/integral operator

$$\mathcal{F}\left[{}_{-\infty}D_t^{\alpha}f(t)\right] = (j\omega)^{\alpha}\mathcal{F}[f(t)], \qquad (8)$$

where α can be either positive or negative real number.







The advantages of fractional order models

They depict the real world objects more accurately;
Variable stability region;
etc...

G.W. Scott Blair (1950)

“We may express our concepts in Newtonian terms if we find this convenient but, if we do so, we must realize that we have made a translation into a language which is foreign to the system which we are studying.”

Integer-Order Calculus

Fractional-Order Calculus

Dr. Richard L. Magin, ICCC12

Zhuo LI 2013

Why many integer order models depict real world objects well enough?

The “fractance” of many objects are usually very low.

Why integer order models are more popular?

Complexity of solutions to FO differential equations;

Ignorance of the existence of fractional order models.

Collection of resources for fractional order control

Fractional Order Controls: A beginner's view
http://mechatronics.ucmerced.edu/sites/mechatronics.ucmerced.edu/files/page/documents/afc-csois-zhuo-2012-5-14.pdf

Chen, Y.; Petras, I.; Xue, D, "Fractional order control - A tutorial" American Control Conference. ACC '09, pp.1397,1411, Jun 10-12, 2009

Podlubny, I, "Fractional-Order Systems and $PI^{\lambda}D^{\mu}$ Controllers", IEEE Transactions on Automatic Control, Vol. 44, No. 1, January 1999

Oustaloup, Alan, etc, The CRONE Team, "From Fractal Robustness to the CRONE Approach", ESAIM: Proc., 1998, Vol. 5, pp. 177-192

Stability of LTI Fractional Order Systems

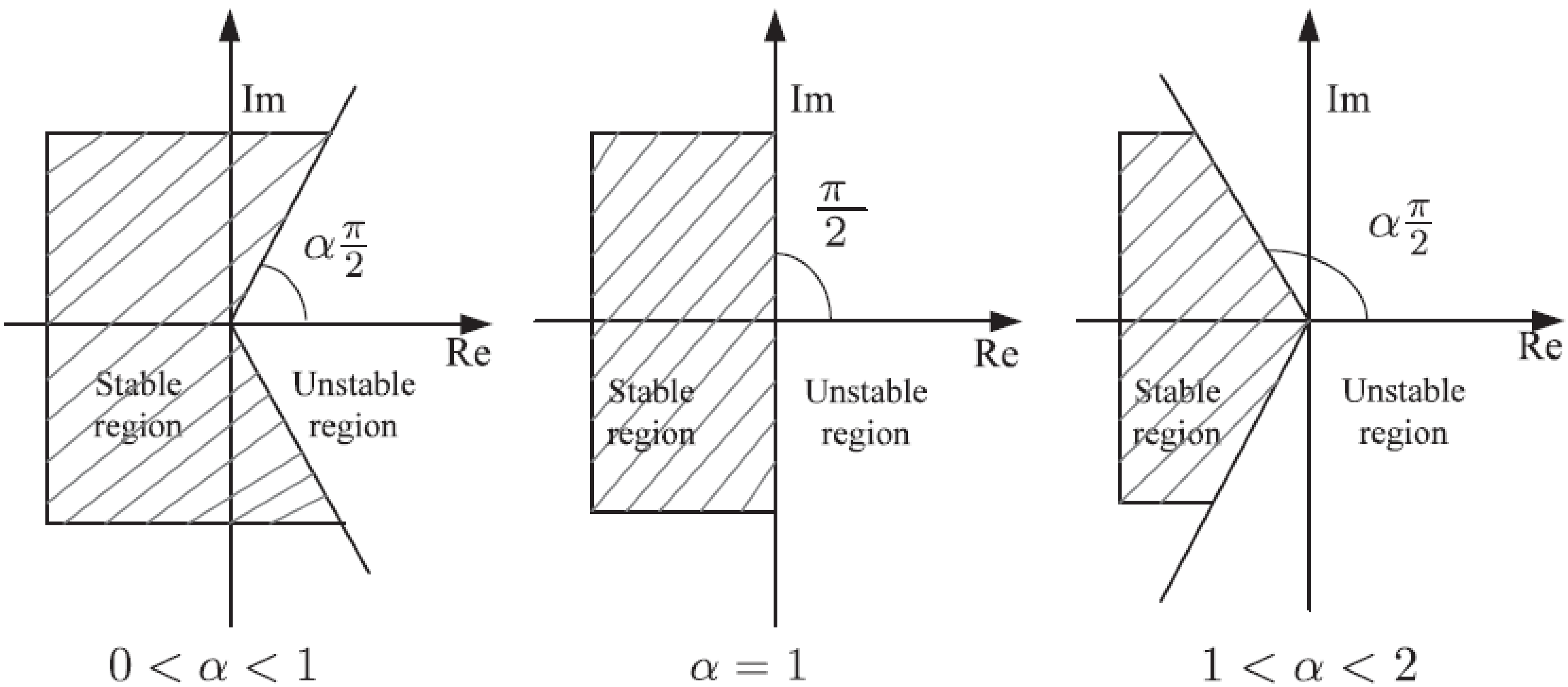


Figure: Stability region of LTI fractional order systems

Four combinations

- IO model, IO controller
- **IO model, FO controller**
- **FO model, IO controller**
- **FO model, FO controller**

TID Controller

Similar to PID control, the proportional compensating unit in TID (“Tilt” integral derivative) control is replaced with a compensator having a transfer function characterized by $1/s^{\frac{1}{n}}$ or $s^{-1/n}$.

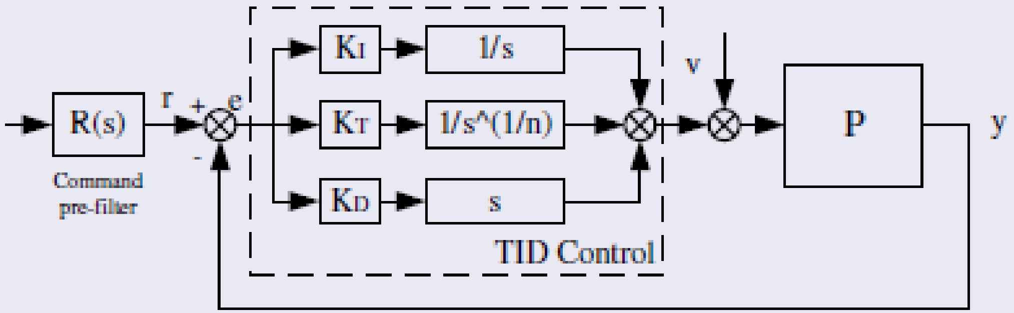
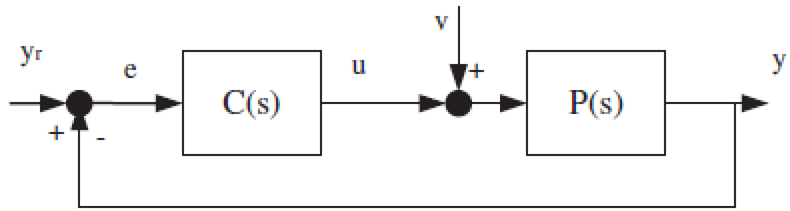
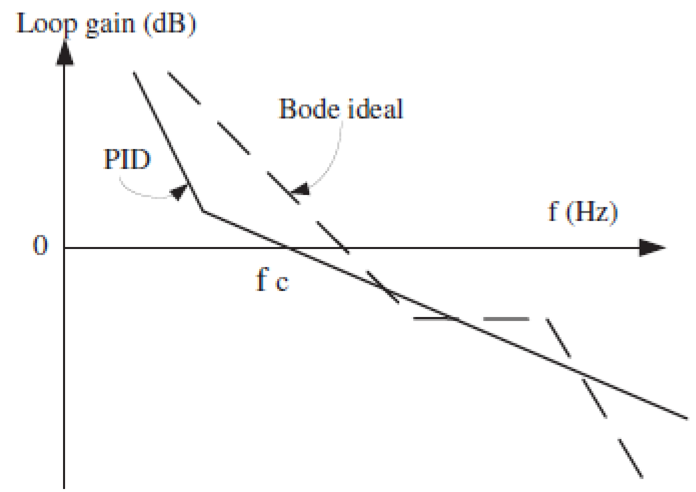


Figure: Block diagram of TID control scheme

Motivation of the TID controller



(a) Block diagram of the classic feedback control system with disturbance



(b) Bode plots for PID controlled plant and the ideal loop response

Figure: Bode plots for PID controlled plant and the ideal loop response

CRONE Controller

CRONE is a French abbreviation for "*Contrôle Robuste d'Ordre Non Entier*" (which means non-integer order robust control).

Fractal Robustness

Used to describe the following two characteristics:

- Iso-damping lines;
- Frequency template.

$\mathrm{PI}^{\lambda}\mathrm{D}^{\mu}$ Controller

$$C(s) = \frac{U(s)}{E(s)} = K_p + T_i s^{-\lambda} + T_d s^{\delta}, \tag{10}$$

Better control performance can be expected due to the introduction of more tuning knobs.

Tuning rule

- No systematic and rigorous method exist as for the conventional PID.
- A simple tuning scheme
  - $K_p > 100/E_t$, where E_t is the static error;
  - Solve T_d, δ, T_i, λ from the characteristic equation withe the FO controller

Fractional Lead-Lag Compensator

$$C_r(s) = C_0 \left(\frac{1 + s/\omega_b}{1 + s/\omega_h} \right)^r, \tag{11}$$

where $0 < \omega_b < \omega_h$, $C_0 > 0$ and $r \in (0, 1)$.

Design concept

- ideal loop transfer function: $(\omega_0/j\omega)^n$ at $|L_0(j\omega_0)| = 1$;
- try to shape the loop transfer function, $L_0(j\omega)$, close the the above shape, i.e. $C(s) = C_r(s)G(s)$, where $G(s) \approx Ms^{-m}/P_0(s)$.

Realization

Stable minimum-phase frequency-domain fitting!

This is the end of session I

Questions?